A New Application of Kamal Transform for Solving Linear Volterra Integral Equations

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Abstract: **In this paper, we used Kamal transform for solving linear Volterra integral equations and some applications are given in order to demonstrate the effectiveness of Kamal transform for solving linear Volterra integral equations.**

Keywords: Linear Volterra integral equation, Kamal transform, Convolution theorem, Inverse Kamal transform.

I. INTRODUCTION

Volterra examined the linear Volterra integral equation of the form [1-5]

$u(x) = f(x) + \lambda \int_0^x k(x,t)u(t)dt$ .... (1)

where the unknown function $u(x)$, that will be determined, occurs inside and outside the integral sign. The kernel $k(x,t)$and the function $f(x)$ are given real-valued functions, and λ is a parameter. The Volterra integral equations appear in many physical applications such as neutron diffusion and biological species coexisting together with increasing and decreasing rates of generating.

The Kamal transform of the function $F(t)$is defined as [6]:

$$K\{F(t)\} = \int_0^\infty F(t)e^{\frac{-t}{v}}dt = G(v), t \geq 0, k_1 \leq v \leq k_2$$

where K is Kamal transform operator.

The Kamal transform of the function $F(t)$ exist if $F(t)$ is piecewise continuous and of exponential order. These conditions are only sufficient conditions for the existence of Kamal tansform of the function $F(t)$.Abdelilah and Hassan used Kamal transform for solving partial differential equations.

The aim of this work is to establish exact solutions for linear Volterra integral equation using Kamal transform without large computational work.

II. KAMAL TRANSFORM OF SOME ELEMENTARY FUNCTIONS [6, 8]:

S.N.	$F(t)$	$K\{F(t)\} = G(v)$
1.	1	v
2.	t	v^2
3.	t^2	$2!\,v^3$
4.	$t^n, n \geq 0$	$n!\,v^{n+1}$
5.	e^{at}	$\frac{v}{1-av}$
6.	$sinat$	$\frac{av^2}{1+a^2v^2}$
7.	$cosat$	$\frac{v}{1+a^2v^2}$
8.	$sinhat$	$\frac{av^2}{1-a^2v^2}$
9.	$coshat$	$\frac{v}{1-a^2v^2}$

III. KAMAL TRANSFORM OF THE DERIVATIVES OF THE FUNCTION$F(t)$ [6, 8, 9]

If $K\{F(t)\} = G(v)$ then

a) $K\{F'(t)\} = \frac{1}{v}G(v) - F(0)$

b) $K\{F''(t)\} = \frac{1}{v^2}G(v) - \frac{1}{v}F(0) - F'(0)$

c) $K\{F^{(n)}(t)\} = \frac{1}{v^n}G(v) - \frac{1}{v^{n-1}}F(0) - \frac{1}{v^{n-2}}F'(0) \ldots\ldots - F^{(n-1)}(0)$

IV. CONVOLUTION OF TWO FUNCTIONS [8]

Convolution of two functions $F(t)$ and $H(t)$ is denoted by $F(t) * H(t)$and it is defined by

$$F(t) * H(t) = F * H = \int_0^t F(x)H(t-x)dx$$
$$= \int_0^t H(x)F(t-x)dx$$

V. CONVOLUTION THEOREM FOR KAMAL TRANSFORMS [8]

If $K\{F(t)\} = G(v)$ and $K\{H(t)\} = I(v)$ then

$$K\{F(t) * H(t)\} = K\{F(t)\}K\{H(t)\} = G(v)I(v)$$

VI. INVERSE KAMAL TRANSFORM

If $K\{F(t)\} = G(v)$ then $F(t)$ is called the inverse Kamal transform of $G(v)$ and mathematically it is defined as $F(t) = K^{-1}\{G(v)\}$

where K^{-1} is the inverse Kamal transform operator.

VII. INVERSE KAMAL TRANSFORM OF SOME ELEMENTARY FUNCTIONS

S. N.	$G(v)$	$F(t) = K^{-1}\{G(v)\}$
1.	v	1
2.	v^2	t
3.	v^3	$\frac{t^2}{2!}$
4.	$v^{n+1}, n \geq 0$	$\frac{t^n}{n!}$
5.	$\frac{v}{1-av}$	e^{at}
6.	$\frac{v^2}{1+a^2v^2}$	$\frac{sinat}{a}$
7.	$\frac{v}{1+a^2v^2}$	$cosat$
8.	$\frac{v^2}{1-a^2v^2}$	$\frac{sinhat}{a}$
9.	$\frac{v}{1-a^2v^2}$	$coshat$

VIII. KAMAL TRANSFORM FOR LINEAR VOLTERRA INTEGRAL EQUATIONS

In this work we will assume that the kernel $k(x,t)$of (1) is a difference kernel that can be expressed by the difference$(x-t)$. The linear Volterra integral equatin (1) can thus be expressed as

$$u(x) = f(x) + \lambda \int_0^x k(x-t)u(t)dt \ \ldots\ldots\ldots\ldots\ldots \quad (2)$$

Applying the Kamal transform to both sides of(2), we have

$$K\{u(x)\} = K\{f(x)\} + \lambda K\{\int_0^x k(x-t)u(t)dt\} \qquad (3)$$

Using convolution theorem of Kamal transform, we have

$$K\{u(x)\} = K\{f(x)\} + \lambda K\{k(x)\}K\{u(x)\}\ldots\ldots\ldots.. (4)$$

Operating inverse Kamal transform on both sides of(4), we have

$$u(x) = f(x) + \lambda K^{-1}\{K\{k(x)\}K\{u(x)\}\}\ldots\ldots\ldots.. \quad (5)$$

which is the required solution of (2).

IX. APPLICATIONS

In this section, some applications are given in order to demonstrate the effectiveness of Kamal transform for solving linear Volterra integral equations.

A. *Application:1* Consider linear Volterra integral equation with $\lambda = -1$

$$u(x) = x - \int_0^x (x-t)\, u(t)dt\ldots\ldots\ldots\ldots (6)$$

Applying the Kamal transform to both sides of(6), we have

$$K\{u(x)\} = v^2 - K\left\{\int_0^x (x-t)\, u(t)dt\right\}\ldots\ldots\ldots\ldots (7)$$

Using convolution theorem of Kamal transform on (7), we have

$$K\{u(x)\} = \frac{v^2}{1+v^2}\ldots\ldots\ldots\ldots (8)$$

Operating inverse Kamal transform on both sides of(8), we have

$$u(x) = K^{-1}\left\{\frac{v^2}{1+v^2}\right\} = sinx\ldots\ldots\ldots\ldots\ldots.. (9)$$

which is the required exact solution of (6).

B. *Application:2* Consider linear Volterra integral equation with $\lambda = -1$

$$u(x) = cosx + sinx - \int_0^x u(t)\, dt\ldots\ldots (10)$$

Applying the Kamal transform to both sides of(10), we have

$$K\{u(x)\} = \frac{v}{1+v^2} + \frac{v^2}{1+v^2} - K\left\{\int_0^x u(t)\, dt\right\}\ldots\ldots\ldots\ldots (11)$$

Using convolution theorem of Kamal transform on(11), we have

$$K\{u(x)\} = \frac{v}{1+v^2}\ldots\ldots\ldots\ldots (12)$$

Operating inverse Kamal transform on both sides of(12), we have

$$u(x) = K^{-1}\left\{\frac{v}{1+v^2}\right\} = cosx\ldots\ldots\ldots\ldots\ldots.. (13)$$

which is the required exact solution of (10).

C. *Application:3* Consider linear Volterra integral equation with $\lambda = 1$

$$u(x) = 1 - x + \int_0^x (x-t)u(t)\, dt\ldots\ldots (14)$$

Applying the Kamal transform to both sides of(14), we have

$$K\{u(x)\} = v - v^2 + K\left\{\int_0^x (x-t)u(t)\, dt\right\}\ldots. (15)$$

Using convolution theorem of Kamal transformon(15), we have

$$K\{u(x)\} = \frac{v}{1-v^2} - \frac{v^2}{1-v^2}\ldots\ldots\ldots\ldots (16)$$

Operating inverse Kamal transform on both sides of(16), we have

$$u(x) = K^{-1}\left\{\frac{v}{1-v^2}\right\} - K^{-1}\left\{\frac{v^2}{1-v^2}\right\}$$

$$= coshx - sinhx = e^{-x}\ldots\ldots\ldots\ldots\ldots (17)$$

which is the required exact solution of (14).

D. *Application:4* Consider linear Volterra integral equation with $\lambda = -1$

$u(x) = 1 - \int_0^x (x-t)u(t)\,dt$…… (18)

Applying the Kamal transform to both sides of(18), we have

$K\{u(x)\} = v - K\{\int_0^x (x-t)u(t)\,dt\}$…. (19)

Using convolution theorem of Kamal transform on(19), we have

$K\{u(x)\} = \frac{v}{1+v^2}$………… (20)

Operating inverse Kamal transform on both sides of(20), we have

$u(x) = K^{-1}\left\{\frac{v}{1+v^2}\right\} = cosx$……….. (21)

which is the required exact solution of (18).

E. Application:5 Consider linear Volterra integral equation with $\lambda = 1$

$u(x) = 1 - \frac{x^2}{2} + \int_0^x u(t)\,dt$……(22)

Applying the Kamal transform to both sides of (22), we have

$K\{u(x)\} = v - v^3 + K\{\int_0^x u(t)\,dt\}$…. (23)

Using convolution theorem of Kamal transform on(23), we have

$K\{u(x)\} = v + v^2$………… (24)

Operating inverse Kamal transform on both sides of(16), we have

$u(x) = K^{-1}\{v\} + K^{-1}\{v^2\}$

$= 1 + x$…………… (25)

which is the required exact solution of (22).

X. CONCLUSION

In this paper, we have successfully developed the Kamal transform for solving linear Volterra integral equations. The given applications showed that the exact solution have been obtained using very less computational work and spending a very little time. The proposed scheme can be applied for other linear Volterra integral equations and their system.

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