# Position Control of Quadrotor using Sliding Mode Technique

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***Abstract*—Quadrotor is a type of unmanned aerial vehicle (UAV), that has abilities such as vertical take-off and landing, hover capability, high maneuverability and agility. This work aims to control a quadrotor to reach a desired position usingsliding mode technique. Modelling of the quadrotor is done by Euler-Legrangian method. Simulation results shows the performance of the quadrotor is satisfactory using this technique.**

***Keywords*—quadrotor, backstepping, sliding mode technique**

## I. INTRODUCTION

The technology of Unmanned Aerial Vehicle(UAV) has grown in the past decade and this field has application in the areas such as remote sensing, surveillance, transport, rescue, remote inspection and photography. Quadrotor is a type of UAV and it has many abilities such as vertical take-off and landing, hover capability, high maneuverability and agility. A quadrotor can hover in space and have six degrees of freedom. Also, it possesses more advantages than standard helicopters in terms of small size, efficiency and safety.

Many control techniques have been developed for the control of quadrotor including back-stepping, sliding mode control, Robust control etc.In [1] a control law was synthesized by sliding mode control based on the back-stepping approach. Linear control techniques like PID and LQR techniques are used in [2], [7] for the control of quadrotor. A new nonlinear control technique, back-stepping like feedback linearization is developed in [3] for the position control and stabilize the quadrotor. Based on the developed nonlinear dynamic equations of a quadrotor (named as Qball-X4) UAV (Unmanned Aerial Vehicle), attitude and trajectory tracking control designs based on an inner/outer loop control structure has been proposed in [4]. An integral back-stepping approach was applied into an autonomous flight of the quadrotor system including indoor experiments and a back-stepping control was used to stabilize the quadrotor's attitude system [5]. Nested saturation control algorithm based on the Lyapunov analysis was proposed to track the desired trajectory including experimental tests [6].

## II. QUARDROTOR MODELLING

The quadrotor model is derived using Euler-Lagrangian method. Let, $q = (x, y, z, \phi, \theta, \psi) \in R^6$ be the generalized coordinates where $\xi = (x, y, z) \in R^3$ denotes the absolute position of the mass of the quadrotor relative to a fixed inertial frame. Euler angles, which are the orientation of the quadrotor, are expressed by $\eta = (\phi, \theta, \psi) \in R^3$ where φ is the roll angle around the x axis, θ is the pitch angle around the y axis, and ψ is the yaw angle around the z axis. It is assumed that the Euler angles are bounded as:

$$\text{Roll}: -\frac{\pi}{2} \le \phi \le \frac{\pi}{2}$$

$$\text{Pitch}: -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$

$$\text{Yaw}: -\pi \le \psi \le \pi \tag{1}$$

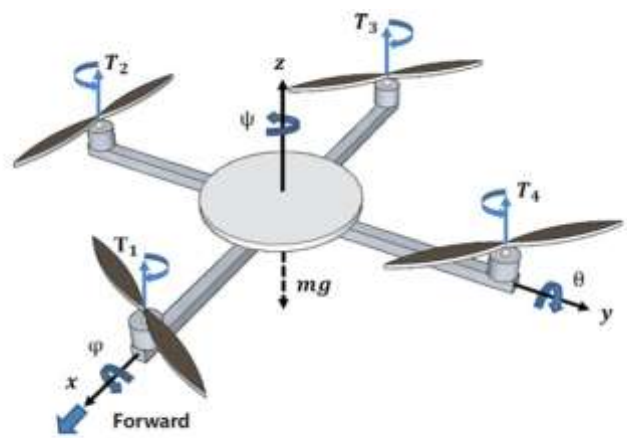


Fig. 1 Quadrotor configuration

The model of quadrotor can be partitioned into translational and rotational coordinates.The translational kinetic energy of the quadrotor is$T_{trans} = \frac{1}{2} m \dot{\xi}^T$, where m is the mass of quadrotor. The rotational kinetic energy is $T_{rot} = \frac{1}{2} I \dot{\eta}^T$. The matrix I is the moment of inertia matrix for the full-rotational kinetic energy of the quadrotor which is expressed directly in terms of the generalized coordinates η. The potential energy is $U = mgz$, where g is the acceleration due to gravity.

Then, the Lagrangian model can be defined as:

$$L(q, \dot{q}) = T_{trans} + T_{rot} - U$$

$$= \frac{1}{2} m \dot{\xi}^T + \frac{1}{2} I \dot{\eta}^T - mgz \tag{2}$$

The full quadrotor dynamics can be obtained by Euler Legrangian equations with external force, $F = (F_\xi, \tau)$as follows,

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = (F_\xi, \tau) \tag{3}$$

where, $F_\xi$is the translational force applied to the quadrotor and τ is the roll, pitch, and yaw moments.The translational force$F_\xi = RF_R$ is where R is the rotation matrix and $F_R$ is the translational force in the inertial frame. The rotation matrix R equals to $R = R_z R_y R_x$which is the orientation of the quadrotor. $F_R$ is defined by

$$F_R = \begin{bmatrix} 0 \\ 0 \\ U_1 \end{bmatrix} \tag{4}$$

where $U_1$is the main thrust. The main thrust is the summation of thrust moments as follows:

$$U_1 = T_1 + T_2 + T_3 + T_4 \tag{5}$$

where $T_i$ is the thrust moment. The thrust moment for each BLDC motor is:

$$T_i = b\omega_i{}^2 \tag{6}$$

where b is a positive constant that denotes the thrust factor of propeller and $\omega_i$ is the angular velocity of the BLDC motor $i$.

Then the control vectors can be defined as,

$$\tau = \begin{bmatrix} U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} = \begin{bmatrix} bl(\omega_4{}^2 - \omega_2{}^2) \\ bl(\omega_3{}^2 - \omega_1{}^2) \\ d(\omega_1{}^2 + \omega_3{}^2 - \omega_2{}^2 - \omega_4{}^2) \end{bmatrix} \tag{7}$$

The Euler–Lagrangian equation can be partitioned into the translational and rotational co-ordinates. Thus, the final model of the quadrotor can be expressed as

$$m\ddot{\xi} = -\begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} + F_\xi \tag{8}$$

$$I\ddot{\eta} + \frac{d}{dt}\{I\}\dot{\eta} - \frac{1}{2}\frac{\partial}{\partial \eta}(\dot{\eta}^T I \dot{\eta}) = \tau \tag{9}$$

The Coriolis–centripetal vector is defined by

$$V(\eta, \dot{\eta}) = \dot{I}\dot{\eta} - \frac{1}{2}\frac{\partial}{\partial \eta}(\dot{\eta}^T I \dot{\eta}) \tag{10}$$

Then,

$$I\ddot{\eta} + V(\eta, \dot{\eta}) = \tau \tag{11}$$

$V(\eta, \dot{\eta})$ can be written as

$$V(\eta, \dot{\eta}) = \left(\dot{I} - \frac{1}{2}\frac{\partial}{\partial t}(\dot{\eta}^T I)\right)\dot{\eta}$$

$$= C(\eta, \dot{\eta})\dot{\eta} \tag{12}$$

where,$C(\eta, \dot{\eta})$ is referred to as the Coriolis term.

$$m\ddot{\xi} = F_\xi - mge_3 \tag{13}$$

$$I\ddot{\eta} = \tau - C(\eta, \dot{\eta})\dot{\eta} \tag{14}$$

Finally, the model of the quadrotor in terms of translation (x, y, z) and rotation (φ, θ, ψ) can be written as follows:

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} c_\phi s_\theta c_\psi + s_\phi s_\psi \\ c_\phi s_\theta s_\varphi - s_\phi c_\psi \\ c_\phi c_\theta \end{bmatrix} \frac{U_1}{m} + \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix} \tag{15}$$

$$\begin{bmatrix} \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = f(\phi, \theta, \psi) + g(\phi, \theta, \psi)\tau_U \tag{16}$$

where $c_\phi$ is cos φ, $s_\phi$ is sin φ, and

$$f(\phi, \theta, \psi) = \begin{bmatrix} \dot{\theta}\dot{\psi}\left(\frac{I_y - I_z}{I_x}\right) - \frac{J_x}{I_x}\dot{\theta}\Omega \\ \dot{\phi}\dot{\psi}\left(\frac{I_z - I_x}{I_y}\right) + \frac{J_x}{I_y}\dot{\phi}\Omega \\ \dot{\phi}\dot{\theta}\left(\frac{I_x - I_y}{I_z}\right) \end{bmatrix} \tag{17}$$

$$g(\phi, \theta, \psi) = \begin{bmatrix} \frac{1}{I_x} & 0 & 0 \\ 0 & \frac{1}{I_y} & 0 \\ 0 & 0 & \frac{1}{I_z} \end{bmatrix} \tag{18}$$

The control inputs $U_1, U_2, U_3, U_4$ can be defined as,

$$U_1 = b(\omega_1{}^2 + \omega_2{}^2 + \omega_3{}^2 + \omega_4{}^2) \tag{19}$$

$$\tau_U = \begin{bmatrix} U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} bl(\omega_4^2 - \omega_2^2) \\ bl(\omega_3^2 - \omega_1^2) \\ d(\omega_1^2 + \omega_3^2 - \omega_2^2 - \omega_4^2) \end{bmatrix} \tag{20}$$

where $U_1$ is essentially used to make the altitude reach the desired value, $U_2$ is used to control the roll and horizontal displacement, $U_3$ is used to control the pitch and vertical movement, $U_4$ is used to set the yaw displacement, $I_{x,y,z}$ are the moment of inertia of quadrotor, $J_r$ is the moment of inertia of rotor, and $\Omega = \omega_1 - \omega_2 + \omega_3 - \omega_4$ .

The model equation (15), (16) can be converted to state space as,

$$x_1 = \phi,$$

$$x_2 = \dot{x_1} = \dot{\phi},$$

$$x_3 = \theta,$$

$$x_4 = \dot{x_3} = \dot{\theta},$$

$$x_5 = \psi,$$

$$x_6 = \dot{x_5} = \dot{\psi},$$

$$x_7 = z,$$

$$x_8 = \dot{x_7} = \dot{z},$$

$$x_9 = x,$$

$$x_{10} = \dot{x_9} = \dot{x},$$

$$x_{11} = y,$$

$$x_{12} = \dot{x_{11}} = \dot{y}$$

$$f(X,U) = \begin{pmatrix} x_2 \\ x_4x_6a_1 + x_4a_2\Omega + b_1U_2 \\ x_4 \\ x_2x_6a_3 + x_2x_4\Omega + b_1U_2 \\ x_6 \\ x_4x_2a_5 + b_3U_4 \\ x_8 \\ -g + (\cos x_1 \cos x_3)\frac{1}{m}U_1 \\ x_{10} \\ u_x\frac{1}{m}U_1 \\ x_{12} \\ u_y\frac{1}{m}U_1 \end{pmatrix} \quad (21)$$

where,

$$a_1 = (I_y - I_z)/I_x$$

$$a_2 = -\frac{J_r}{I_x}$$

$$a_3 = (I_z - I_x)/I_y$$

$$a_4 = \frac{J_r}{I_y}$$

$$a_5 = (I_x - I_y)/I_z$$

$$b_1 = l/I_x$$

$$b_2 = l/I_y$$

$$b_3 = 1/I_z$$

$$u_x = \cos x_1 \sin x_3 \cos x_5 + \sin x_1 \sin x_5$$

$$u_y = \cos x_1 \sin x_3 \sin x_5 - \sin x_1 \cos x_5$$

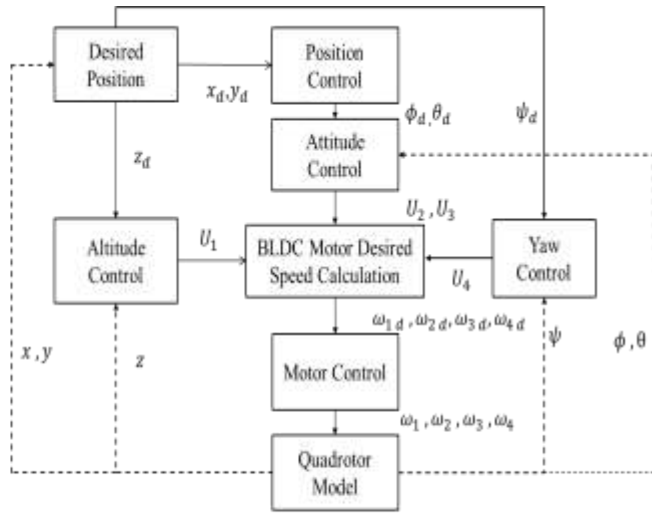


Fig. 2 Control scheme of quadrotor

Figure 2 shows the control schematic of the quadrotor. x, y, z are the translational coordinates of the quadrotor model. $x_d, y_d, z_d$ are the desired translational coordinates. The z coordinate of the quadrotor model is compared with the desired z coordinate and a control vector $U_1$ is computed using sliding mode technique. But in-order to make a movement along x and y direction, its attitude must be changed, that is roll and pitch angles. From $x_d, y_d$ the desired Euler angles $\phi_d, \theta_d$ are calculated using position control and it is compared with quadrotor model $\phi$ and $\theta$, then control vector $U_2, U_3$ are computed using sliding mode technique. Similarly, the yaw angle of the quadrotor model is compared with the desired yaw angle and a control vector $U_4$ is computed using sliding mode technique.

## III. CONTROLLER DESIGN

Non-linear controller of the quadrotor includes attitude control altitude control. The main objective of designed controller is to ensure that the position of quadrotor $(x, y, z)$ tracks the desired trajectory $(x_d, y_d, z_d)$. The attitude controller is to design control inputs $U_2 \sim U_4$. The altitude controller provides the control vector $U_1$ to reach the quadrotor to a particular altitude.

### A) *Attitude Controller*

Attitude control of the quadrotor is the crucial part for stabilization and tracking control problem. $S_2$ surface is used instead of $z_2$. Let consider the error as,

$$z_1 = x_{1d} - x_1 \quad (22)$$

Let the Lyapnov function $V(z_1)$ can be taken as positive definite and the time derivative of Lyapnov function as negative semi definite.

$$V(z_1) = \frac{1}{2}z_1^2 \quad (23)$$

$$\dot{V}(z_1) = z_1(\dot{x_{1d}} - x_2) \quad (24)$$

To make $\dot{V}(z_1)$ negative semi definite, a virtual control input $x_2$ can be introduced as,

$$x_2 = \dot{x_{1d}} + \alpha_1 z_1, \text{ with } \alpha_1 > 0 \quad (25)$$

Let,

$$z_2 = x_2 - \dot{x_{1d}} - \alpha_1 z_1 \quad (26)$$

Since $z_2$ is considering as sliding surface $s_2$,

$$s_2 = x_2 - \dot{x_{1d}} - \alpha_1 z_1 \quad (27)$$

Considering the Lyapnov function including $s_2$,

$$V(z_1, s_2) = \frac{1}{2}(z_1^2 + s_2^2) \quad (28)$$

By proportional rate reaching law,

$$\dot{s_2} = -k_1 \dot{sign}(s_2) - k_2 s_2$$

$$= \dot{x_2} - \ddot{x_{1d}} - \alpha_1 \dot{z_1}$$

$$= a_1 x_4 x_6 + a_2 x_4 \Omega + b_1 U_2 - \ddot{x_{1d}} + \alpha_1(z_2 + \alpha_1 z_1) \quad (29)$$

By back-stepping approach the control $U_2$ can be extracted as: $U_2 = \frac{1}{b_1}(-a_1 x_4 x_6 - a_2 x_4 \Omega - \alpha_1^2 z_1 - k_1 sign(s_2) - k2s2$ (30)

Similarly, the control vectors $U_3, U_4$ can be calculated with the errors,

$$z_3 = x_{3d} - x_3 \quad (31)$$

$$s_3 = x_4 - \dot{x_{3d}} - \alpha_2 z_3 \quad (32)$$

$$z_5 = x_{5d} - x_5 \quad (33)$$

$$s_4 = x_6 - \dot{x_{5d}} - \alpha_3 z_5 \quad (34)$$

The obtained control vectors are

$$U_3 = \frac{1}{b_2}(-a_3 x_2 x_6 - a_4 x_2 \Omega - \alpha_2^2 z_3 - k_3 sign(s_3) - k_4 s_3)$$

$$U_4 = \frac{1}{b_3}(-a_5 x_2 x_4 - \alpha_3^2 z_5 - k_5 sign(s_4) - k_6 s_4) \quad (35)$$

### B) *Altitude Controller*

The altitude of the system is controlled by the vertical force control input $U_1$. Let the altitude error $z_7$ and $z_8$ as follows:

$$z_7 = x_{7d} - x_7 \quad (36)$$

$$z_8 = x_8 - \dot{x_{7d}} + \alpha_7 z_7 \quad (37)$$

Let, $z_8$ can be taken as the sliding surface $s_8$,

$$s_8 = x_8 - \dot{x_{7d}} + \alpha_7 z_7 \quad (38)$$

The Lyapnov candidate $V(z_7, s_8)$ can be defined as:

$$V(z_7, s_8) = \frac{1}{2}({z_7}^2 + {s_8}^2) \quad (39)$$

By proportional rate reaching law,

$$\dot{s_8} = -k_7 \dot{sgn}(s_8) - k_8 s_8$$

$$= x_8 - \dddot{x_{7d}} - \alpha_7 \dot{z_7}$$

$$= (\cos x_1 \cos x_3)\frac{U_1}{m} - g + \alpha_7(z_8 + \alpha_7 z_7) \quad (40)$$

By back-stepping approach the control vector $U_1$ can be calculated as,

$$U_1 = \frac{m}{\cos x_1 \cos x_3}[-k_8 s_8 + g - \alpha_7 z_8 - \alpha_7^2 z_7 - k_7 sgn(s_8)] \quad (41)$$

## IV. SIMULATION RESULTS

The control parameters are chosen to be low values as

$\alpha_1 = \alpha_2 = \alpha_3 = \alpha_7 = \alpha_8 = 3, k_1 = k_3 = k_5 = 1, k_2 = k_4 = k_6 = 2$

Here, the desired position is given as [10, 10, 10].

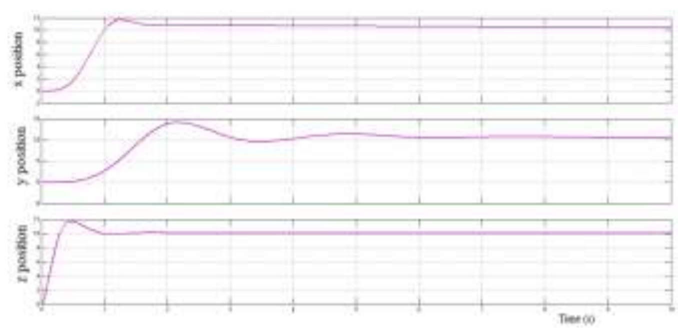


Fig. 3 Response of positions with sliding mode control

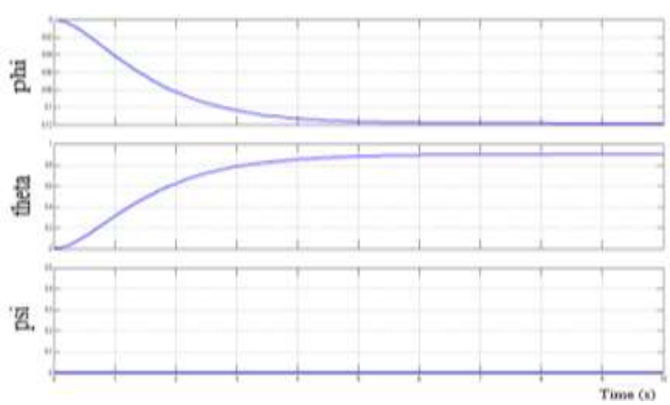


Fig. 4 Response of the Euler angles with sliding mode control

From the figure 3, it is clear that the quadrotor tracks the desired position [10, 10, 10] with the Euler angles shown in figure 4.

TABLE I

Analysis of the response

| Position | Rise time | Peak overshoot | Settling time |
|---|---|---|---|
| X | 1 s | 15.2 % | 10 s |
| Y | 1.55 s | 31.03 % | 10 s |
| Z | 0.29 s | 15.96 % | 2 s |

## V. CONCLUSION

A non-linear control technique, sliding mode technique has been used to track the desired position of the quadrotor.Modelling of the quadrotor has done using Euler Lagrangian equation and stability of the designed controller is verified by the Lyapnov stability theorem. Using the derived control vector $U_1$ , the quadrotor can reach a desired altitude and by the control vectors $U_2, U_3, U_4$ the quadrotor can reach the desired position. The simulation results show the quadrotor tracks the desired position.

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