# Vibration Analysis of Simple Rotor with Transverse Crack

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***Abstract*** **- To examine the vibration characteristics of cracked rotor, a mathematical model for cracked shaft is developed and solved using matlab and simulink. The results are validated by experimentation and using Ansys. A shaft with disk mounted between two bearings is used. An artificial crack is introduced to detect the vibration characteristics of the rotor. Initially, the signature data was obtained with no crack using spectrum analyzer. Later, crack was introduced with depths ranging from 2 mm to 8 mm with four steps on the rotor of 15 mm diameter. On observing the peak amplitudes of right and left bearings, a difference in the values was observed. This indifference in the magnitude of the peak amplitude is because of the combined effect of coupling and position of the crack. A simplified model is solved using ANSYS and the harmonic analysis was performed for different combinations of crack depths and results were obtained. The results obtained by using the three analysis methods gives almost same peak amplitudes.**

***Key words:*** **Cracked rotor, Spectrum Analyser, Harmonic analysis , Vibration characteristics, peak amplitudes.**

## I. INTRODUCTION

Flaws in the components of a structure can influence the dynamic behavior of the whole structure. It is well known from the literature that one form of damage that can lead to catastrophic failure if undetected is fatigue cracking of the structure elements. The recognition of the vibration effects of cracks is important in practice since vibration monitoring has revealed a great potential for investigation of cracks in the last three decades. Usually the physical dimensions, boundary conditions, the material properties of the structure play important role for the determination of its dynamic response. Their vibrations cause changes in dynamic characteristics of structures. In addition to this presence of a crack in structures modifies its dynamic behavior.

A.K. Darpe, K. Gupta, A. Chawla, et.al. [1] studied the effect of the presence of the single transverse crack on the response of the rotor The results of the study are useful in diagnosing fatigue cracks in real rotors, which invariably have some asymmetry. A.K. Darpe, K. Gupta, A. Chawla, [2] studied the coupling between longitudinal, lateral and torsional vibrations together for a rotating cracked shaft. Coupled torsional–longitudinal vibrations for a cracked rotor that has not been reported earlier and coupled torsional–bending vibrations with a breathing crack model have been studied. An attempt has been made to reveal crack specific signatures by using additional external excitations. Qing He, Huichun Peng, Pengcheng Zhai, Yaxin Zhen [3] et.al. have taken the angular acceleration into the consideration for the modeling of equations of coupling vibration in rotational operation. The effects of angular acceleration on the amplitude of both lateral and torsion vibration of the breathing cracked rotor are studied. Yan liLin , Fule Chu et.al.[4] analysed steady responses and indicated that the combined frequencies of the rotating speed and the torsional excitation in the transversal response and the frequency of the torsional excitation in the longitudinal response can be used to detect the slant crack on the shaft of the rotor system. Rugerri Toni Liong , Carsten Proppe [5] et.al. proposed a method for the evaluation of the stiffness losses in the cross-section that contains the crack. This method is based on a cohesive zone model (CZM) instead of linear elastic fracture mechanics (LEFM).

The following aspects of the crack greatly influence the dynamic response of the structure.

I. The position of crack
II. The depth of crack
III. The orientation of crack
IV. The number of cracks

## II. PROBLEM DESCRIPTION AND ANALYSIS PROCEDURE

A rotor having Diameter D is considered for analysis. A transverse surface crack of depth a, is assumed at the midspan of the rotor. The crack is assumed to have a straight edge. The objective is to determine the effect of crack on the amplitude of vibrations wrt the frequency. A mathematical model is developed and it is solved by writing mat lab code along with the simulink model. The analysis is performed using Ansys and the results are validated by conducting experimentation using FFT analyser.

*Mathematical model for Cracked Rotor*

Fracture mechanics concept is used to determine stress intensity factors (SIF ) at the crack and these SIFs are used in solving the equations of motion. The crack model and geometry are shown in Fig.1 and Fig.2.

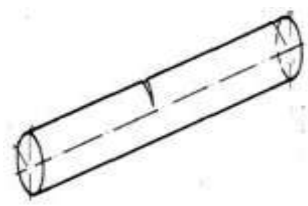


Fig.1 Transverse crack on rotor

The stress intensity factor ($K_1^I$) on the crack is given by equation (1)

$$K_{Q_\xi}{}^{\mathrm{I}} = \sigma_\xi \sqrt{\pi\alpha} F \qquad (1)$$

$$where \quad \sigma_\xi = \frac{\left(\frac{Q_\xi L}{4}\right)\left(\frac{\alpha'}{2}\right)}{I}$$

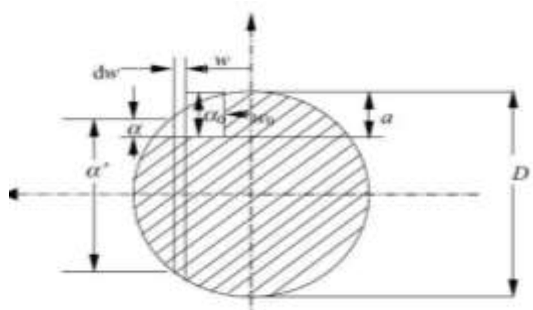



Fig.2 Crack Geometry

$$K_{Q_\xi}{}^{\mathrm{I}} = \frac{Q_\xi L \alpha'}{8I} \sqrt{\pi\alpha} F$$

The stress intensity factor in $\xi$ direction is given by equation (2)

$$K_{Q_\eta}{}^{\mathrm{I}} = \sigma_\eta \sqrt{\pi\alpha} F' \qquad (2)$$

Where $\sigma_\eta = \frac{\left(\frac{Q_\eta L}{4}\right)(\omega)}{I}$,

$$K_{Q_\eta}{}^{\mathrm{I}} = \frac{Q_\eta L \omega}{4I} \sqrt{\pi\alpha} F'$$

Where $I = \left(\frac{\pi}{64}\right) d^4$ and functions $F$ and $F'$, which depend on crack parameters $\alpha$ and $\alpha'$, are given as under

$$F = \sqrt{\frac{2\alpha'}{\pi\alpha} \tan\left(\frac{\pi\alpha}{2\alpha'}\right)} \frac{0.923 + 0.199\left[1 - \sin\left(\frac{\pi\alpha}{2\alpha'}\right)\right]^4}{\cos\left(\frac{\pi\alpha}{2\alpha'}\right)}$$

$$F' = \sqrt{\frac{2\alpha'}{\pi\alpha} \tan\left(\frac{\pi\alpha}{2\alpha'}\right)} \frac{0.752 + 2.02\left(\frac{\alpha}{\alpha'}\right) + 0.37\left[1 - \sin\left(\frac{\pi\alpha}{2\alpha'}\right)\right]^3}{\cos\left(\frac{\pi\alpha}{2\alpha'}\right)}$$

Total deflection in both directions (ξ and η):

$$\hat{u}_\xi = \frac{Q_\xi L^3}{48EI} + u_\xi = \frac{Q_\xi L^3}{48EI} + \frac{2}{E} \iint \left(\frac{8Q_\xi L\alpha'\sqrt{\pi\alpha}F}{\pi D^4} + \frac{16Q_\eta L\omega\sqrt{\pi\alpha}F'}{\pi D^4}\right)\left(\frac{8L\alpha'\sqrt{\pi\alpha}F}{\pi D^4}\right) d\alpha d\omega$$

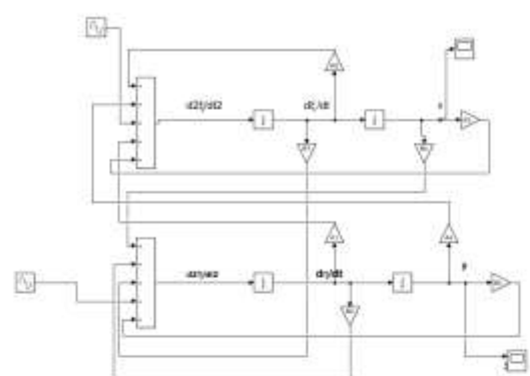
$$\hat{u}_\eta = \frac{Q_\eta L^3}{48EI} + u_\eta = \frac{Q_\eta L^3}{48EI} + \frac{2}{E} \iint \left(\frac{8Q_\xi L\alpha'\sqrt{\pi\alpha}F}{\pi D^4} + \frac{16Q_\eta L\omega\sqrt{\pi\alpha}F'}{\pi D^4}\right)\left(\frac{16L\omega\sqrt{\pi\alpha}F'}{\pi D^4}\right) d\alpha dw$$

The equations of motion are

$$\ddot{\bar{\xi}} - 2\dot{\bar{\eta}} + 2\gamma P_0 \dot{\bar{\xi}} + \bar{\xi}\left({P_\xi}^2 - 1\right) + \dot{\bar{\eta}}\left({P_{\xi\eta}}^2 - 2\gamma P_0\right) = \varepsilon cos\beta - {P_0}^2 cos\tau \text{ and}$$

$$\ddot{\bar{\eta}} - 2\dot{\bar{\xi}} + 2\gamma P_0 \dot{\bar{\eta}} + \bar{\eta}\left({P_\eta}^2 - 1\right) + \dot{\bar{\xi}}\left({P_{\xi\eta}}^2 + 2\gamma P_0\right) = \varepsilon sin\beta - {P_0}^2 sin\tau$$

The above equations of motion are solved by writing matlab code and simulink model and the frequency Vs amplitude plots for different crack depths are obtained.



*Analysis using Ansys:*

A Simple Rotor of length 0.4m length and diameter 0.02 m is considered for analysis. Bearings are modeled at both the ends. Contact pair is used at contact surfaces between shaft and bearing surface . Tetrahedron 10 node 187 element is used for meshing. Outer surface of bearings are constrained and angular velocity of rotating shaft is given as input so that the centrifugal force is considered while performing analysis . Disc weight of 0.8 Kg is applied at 2 cm on the right side of the crack. Vibration analysis is completed for un cracked and different crack depths of 2mm , 4 mm , 6 mm and 8 mm. Frequency Vs amplitude plots are drawn for each case.

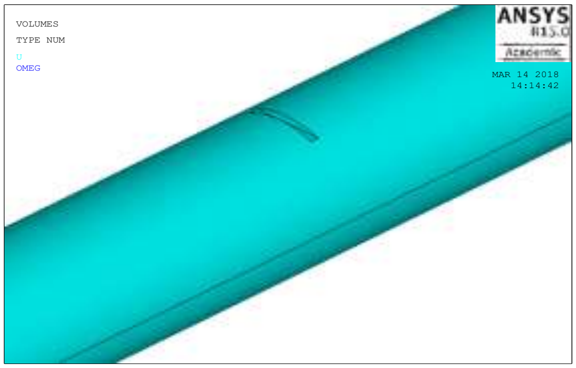



Fig.3 Cracked Shaft Model

*Experimental Procedure:*

A 20mm diameter shaft with a length of 360mm is considered for analysis. The machining operations like turning, facing, finishing and parting off on the lathe machine are completed. After completing all the machining operations the shaft dimensions are shaft diameter 15mm and length of the shaft 350mm. Next step is preparation of crack on the shaft. Preparation of crack with 0.2mm width is difficult task with less cost. But for preparing the crack of different depths we used jewel saw.

Another important aspect is the depth of the crack as depth variations effect the amplitude of vibrations. Five shafts with the same dimensions are considered for experimentation. First sample is shaft is without crack and then second shaft is with the crack depth of 2 mm. The depth of the shaft is measured with the help of tool makers microscope. The remaining three shaft are with 4mm,6mm and 8mm depths. After completing the five shafts with various crack depth with same width , we proceeded to conduct experimentation with the FFT analyser . FFT analyser means fast fourier transformer and it gives the complete information of amplitudes, frequencies, velocity, acceleration with respect to time. For conducting the FFT analyser test ,it requires electric motor, couplings with rubber bushes, two high speed steel ball bearings, table with grooves which the two bearings and weight (800gm).Operational producer for FFT analysers starts with shaft setting. First step is inserting the shaft into the two bearings and adjust the length between the two bearings and place a disc having weight of 800gm at a place of 2cm right from the crack. Next step is to fix the two bearings in the grooves with the help of nuts and bolts. After completing all the arrangements related to shaft setting , experimentation starts with starting the electric motor and regulate the speed slowly up to the fixed speed of 1000rpm. Contactless tachometer is used to observe the speed. The FFT analyser consists of magnetic probe. When using the FFT analyser , the magnetic probe is attached with first bearing and wait for some time then FFT analyser gets the all information about the bearing.

Fig.4 Crack in Rotor Shaft

Fig.5 Experimental Set up

## III. RESULTS

By solving the mathematical model with the aid of simulink model frequency Vs Amplitude plot is drawn for crack depths of 0 mm ( uncracked ) , 2mm , 4mm ,6 mm and 8 mm. Using Ansys , the frequency Vs Amplitude plot is drawn for crack depths of 0mm ( uncracked ) , 2mm , 4mm ,6 mm and 8 mm. Experimental results are also obtained for the above five cases. In experimental analysis , in addition to the frequency Vs amplitude plots , time Vs plot amplitude plots are also taken for different crack depths. The results obtained are shown in figures given below.

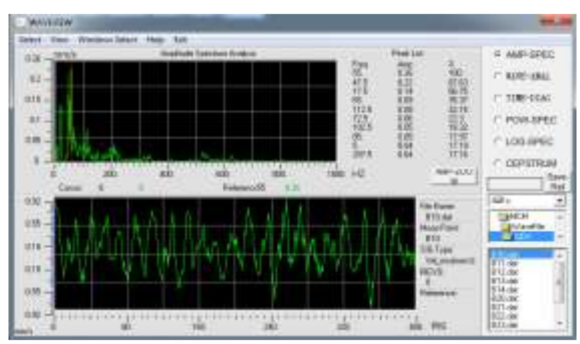


Fig.6 Frequency Vs Amplitude plot and Time Vs Amplitude plot for uncracked shaft

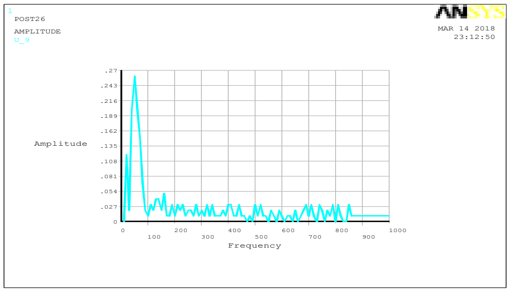



Fig.7 Frequency Vs Amplitude plot and Time Vs Amplitude plot for uncracked shaft (Left bearing )-Using ANSYS

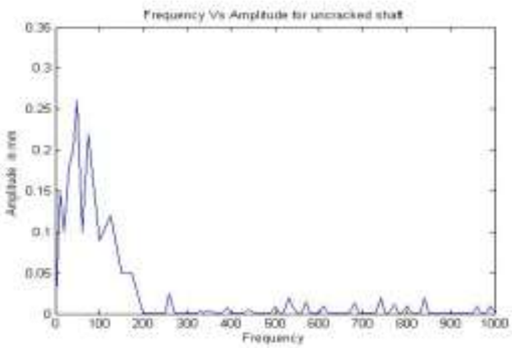



Fig.8 Frequency Vs Amplitude plot and Time Vs Amplitude plot for uncracked shaft (Left bearing )-Using MATLAB code and SIMULINK

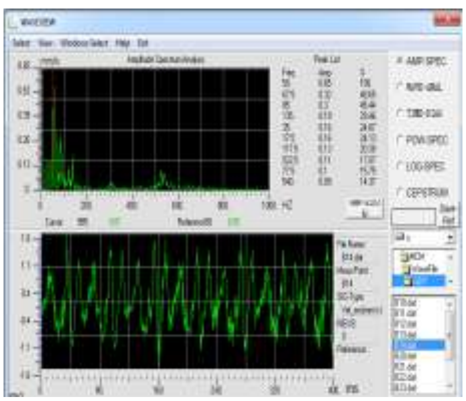


Fig.9 Frequency Vs Amplitude plot for 2mm cracked shaft (Left bearing )-Experimental Analysis

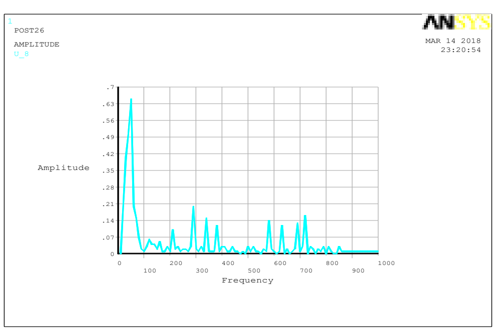



Fig.10 Frequency Vs Amplitude plot for 2mm cracked shaft (Left bearing)-Using ANSYS

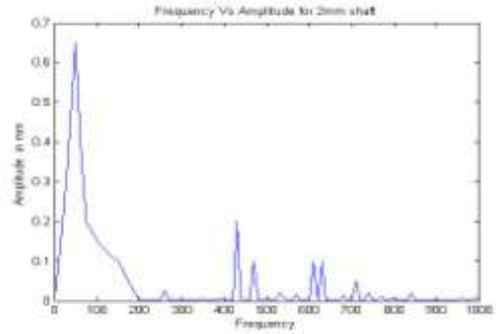



Fig.11 Frequency Vs Amplitude plot for 2mm cracked shaft (Left bearing)-Using MATLAB code and SIMULINK

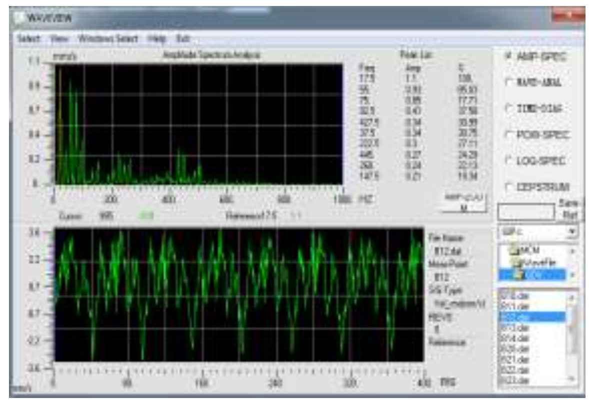


Fig.12 Frequency Vs Amplitude plot for 4mm cracked shaft (Left bearing)-Experimental analysis

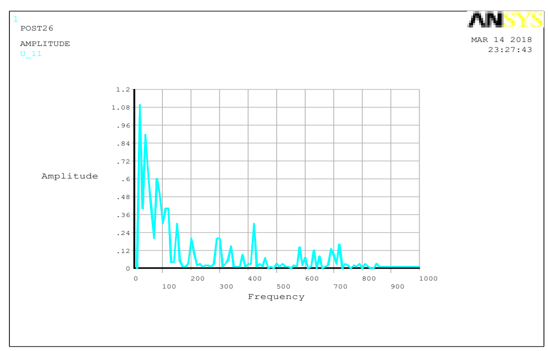



Fig.13 Frequency Vs Amplitude plot for 4mm cracked shaft (Left bearing)-Using ANSYS

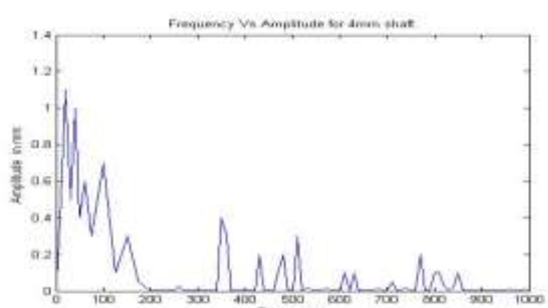



Fig.14 Frequency Vs Amplitude plot for 4mm cracked shaft (Left bearing)-Using MATLAB code and SIMULINK

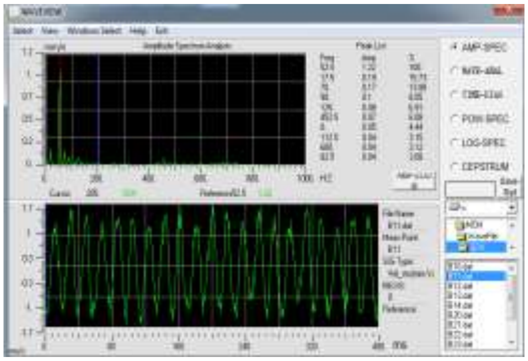


Fig.15 Frequency Vs Amplitude plot for 6mm cracked shaft (Left bearing)-Experimental

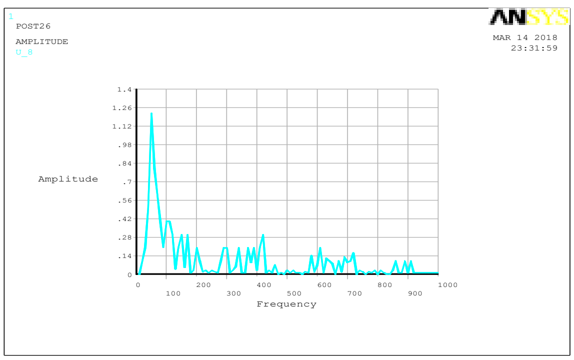



Fig.16 Frequency Vs Amplitude plot for 6mm cracked shaft (Left bearing)-Using ANSYS

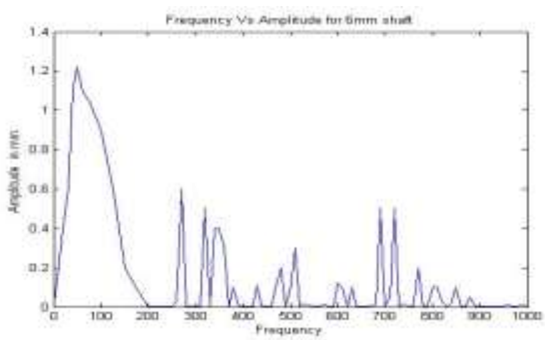



Fig.17 Frequency Vs Amplitude plot for 6mm cracked shaft (Left bearing)-Using MATLAB code and SIMULINK

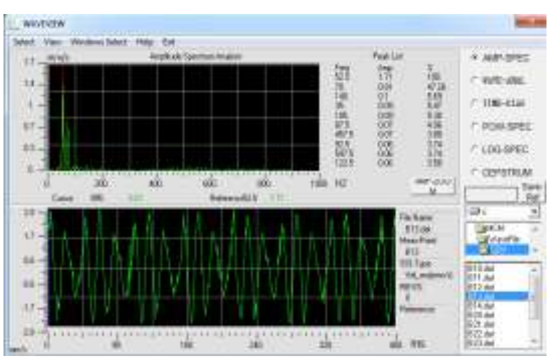


Fig.18 Frequency Vs Amplitude plot for 8 mm cracked shaft (Left bearing)-Experimental analysis

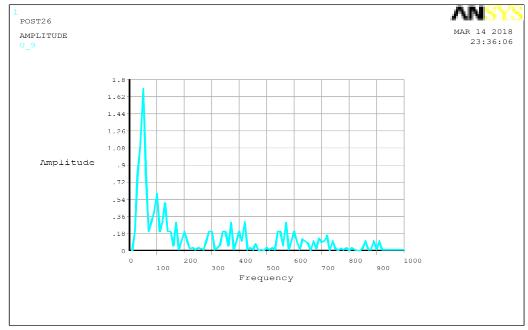



Fig.19 Frequency Vs Amplitude plot for 8 mm cracked shaft (Left bearing)-Using ANSYS

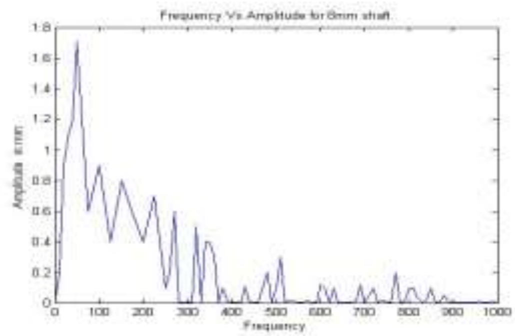



Fig.20. Frequency Vs Amplitude plot for 8 mm cracked shaft (Left bearing)-Using MATLAB code and SIMULINK.

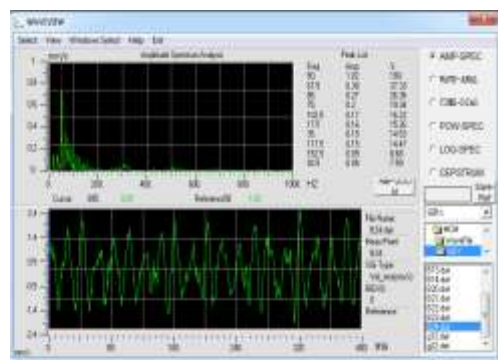


Fig.21. Frequency Vs Amplitude plot for uncracked shaft (Right bearing )-Experimental Analysis.

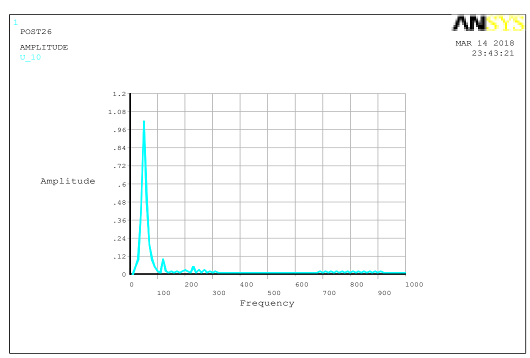



Fig.22.Frequency Vs Amplitude plot for uncracked shaft (Right bearing )-Using ANSYS.

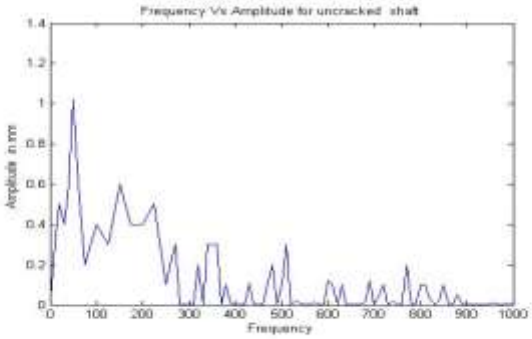



Fig.23. Frequency Vs Amplitude plot for uncracked shaft (Right bearing )-Using MATLAB code and SIMULINK

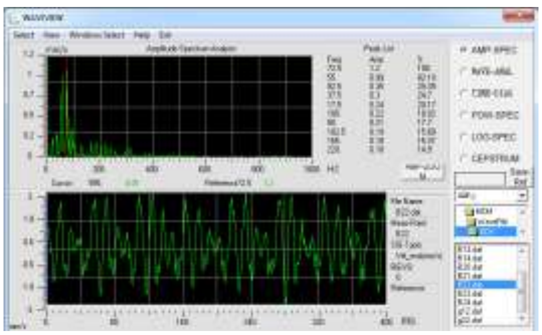


Fig.24. Frequency Vs Amplitude plot for 2mm cracked shaft (Right bearing )-Experimental Analysis

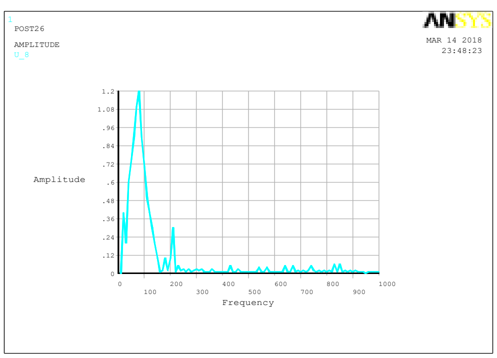



Fig.25. Frequency Vs Amplitude plot for 2mm cracked shaft (Right bearing )-Using ANSYS

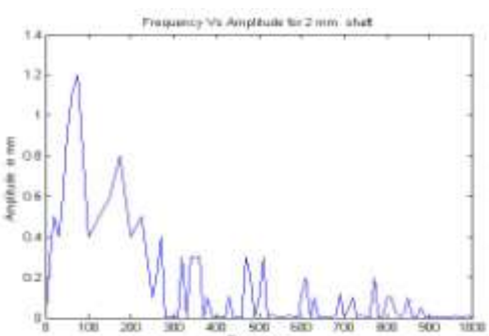



Fig.26.Frequency Vs Amplitude plot for 2mm cracked shaft (Right bearing )-Using MATLAB code and SIMULINK.

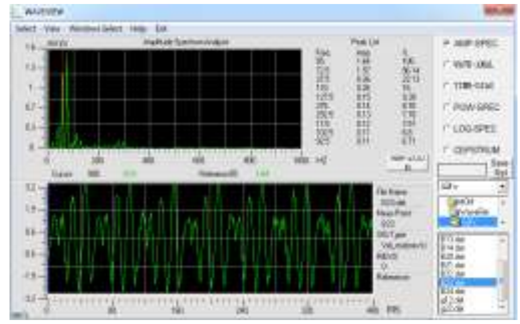


Fig.27.Frequency Vs Amplitude plot for 4mm cracked shaft (Right bearing )-Experimental Analysis.

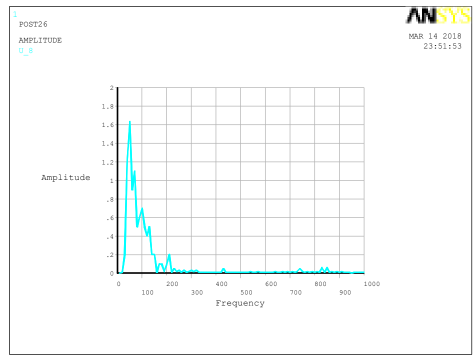



Fig.28. Frequency Vs Amplitude plot for 4mm cracked shaft (Right bearing )-Using ANSYS.

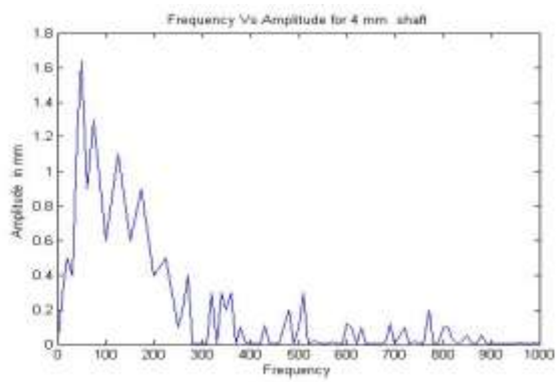



Fig.29.Frequency Vs Amplitude plot for 4mm cracked shaft (Right bearing )-Using MATLAB code and SIMULINK.

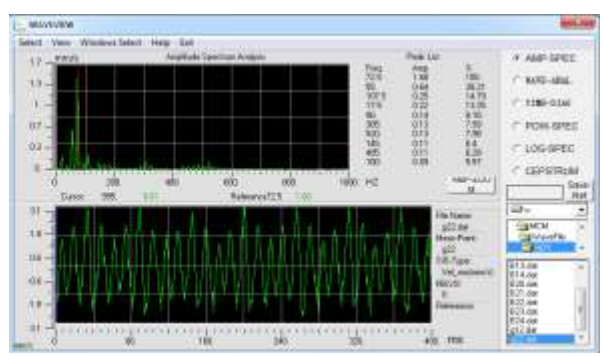


Fig.30.Frequency Vs Amplitude plot for 6 mm cracked shaft (Right bearing)-Experimental Analysis.

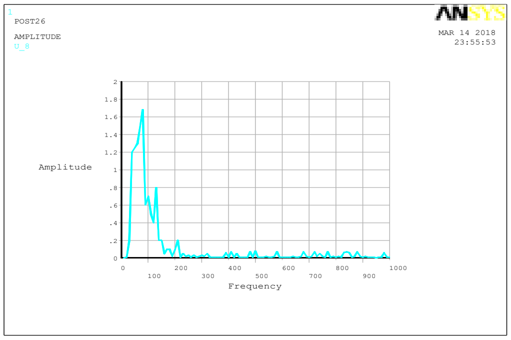



Fig.31. Frequency Vs Amplitude plot for 6 mm cracked shaft (Right bearing)-Using ANSYS.

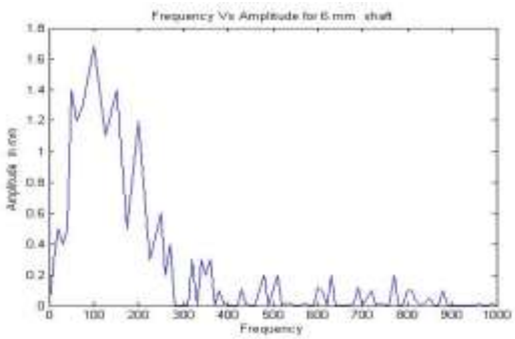



Fig.32.Frequency Vs Amplitude plot for 6 mm cracked shaft (Right bearing)-Using MATLAB code and SIMULINK

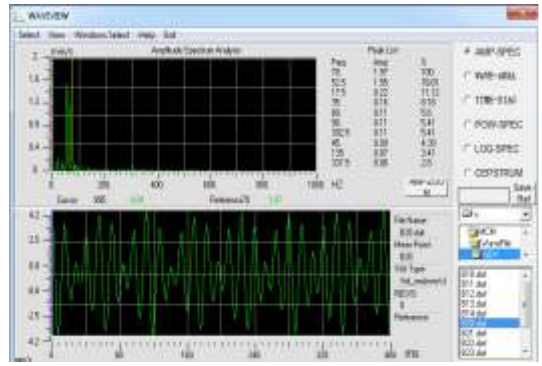


Fig.33. Frequency Vs Amplitude plot for 8 mm cracked shaft (Right bearing)-Experimental Analysis

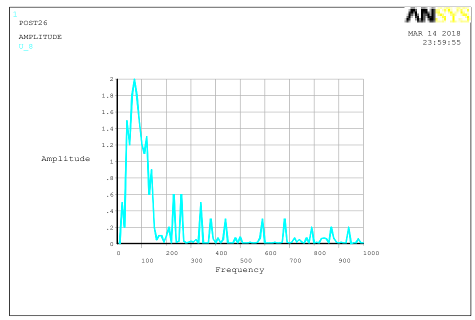



Fig.34. Frequency Vs Amplitude plot for 8 mm cracked shaft (Right bearing)-Using ANSYS

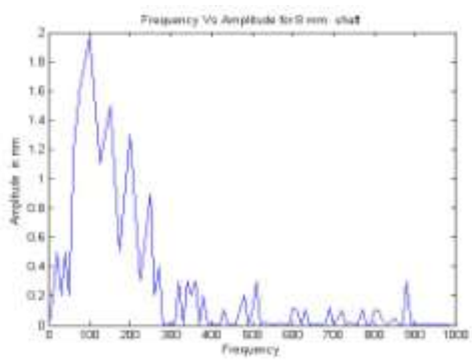



Fig.35. Frequency Vs Amplitude plot for 8 mm cracked shaft (Right bearing )-Using MATLAB code and SIMULINK.

## IV. COMPARISON OF MAXIMUM AMPLITUDE

| S No. | Crack Depth | Max. Amplitudes (Left Bearing) mm | | | Max. Amplitudes (Right Bearing) mm | | |
|---|---|---|---|---|---|---|---|
| | | Mathematical Model | Experimental | Ansys | Mathematical Model | Experimental | Ansys |
| 1 | Without crack | 0.28 | 0.26 | 0.27 | 1.03 | 1.02 | 1.04 |
| 2 | 2mm | 0.63 | 0.65 | 0.66 | 1.21 | 1.2 | 1.22 |
| 3 | 4mm | 1.09 | 1.1 | 1.2 | 1.65 | 1.64 | 1.66 |
| 4 | 6mm | 1.23 | 1.22 | 1.25 | 1.66 | 1.68 | 1.67 |
| 5 | 8mm | 1.69 | 1.71 | 1.68 | 2.01 | 1.97 | 2 |

## V. CONCLUSION

A simple rotor shaft with different crack depths is considered and the vibration characteristics are studied. Initially, mathematical model is solved with the aid of matlab and Simulink. Experimentation is conducted and using the FFT analyzer, frequency Vs amplitude and time Vs amplitude graphs are plotted. The same problem is solved using ANSYS and frequency Vs amplitude graphs are plotted. It is observed that both the analysis methods give approximately same maximum amplitudes. It is also observed that the amplitude of vibrations increases as the crack depth increases.

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