Steady Free Convective Flow through a Vertical Channel with Variable Fluid Properties and Thermal Radiation Effects

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Abstract: **Interaction of variable fluid properties with thermal radiation on free convective flow is investigated using non-linear Rosseland heat diffusion. In particular; the variable fluid properties considered are that the fluid viscosity and thermal conductivity assumed variable status in order to capture more realistic situation. Appropriate similarity variables are defined and used to transform the governing equations from dimensional to dimensionless form and the resulting equations are highly nonlinear due to the presence of variable fluid properties and thermal radiation effects. Semi analytical method of solution; popularly known as Adomian decomposition method (ADM) and computer algebra package were deployed for the solution of the emerging equations capturing the present situation considering the effects of essential physical parameters involved. During the investigation, it was found that an increase in thermal radiation parameter results to corresponding increase in both the fluid's velocity and temperature in the channel. Similarly; an increase in viscosity variation parameter was observed to results tothe increase in the fluid velocity while an increase in thermal conduction parameter was realized to descend both the fluid's velocity and temperature. The results here in have been validated with the published work of Singh and Paul [1] where good agreement was found.**

Keywords: **Free convection; Variable viscosity; Variable thermal conductivity; Thermal radiation; Adomian decomposition method (ADM).**

I. INTRODUCTION

The interaction of free convective flows with thermal radiation through vertical channel has received much attention in recent years due to its importance in many scientific and technological applications (Makinde and Osalusi [2], Shetz and Fuhs [3], and Makinde [4]).

The use of channels in fluid flows are frequently used in designing ventilations, heating of buildings, cooling of electronic components in computers, drying of agricultural grains and foods and so on. Elanbass [5] laid down the pioneering work in fluid dynamics; since then related studies were carried out to this effects; amongst which were that of Miyatake *et al.* [6], Jha [7] and Ostrach [8] in which the walls of the bounding channels were subjected to different thermal conditions.

Thermal radiation is the emission of internal energy of a system in the form of electromagnetic waves to the surrounding environment and this affects its performance and human survival on the earth. In view of its importance for quality control and safety of properties and lives for human existence especially in working condition that requires liberation of heat; several studies were accorded and can be seen in Makinde and Mhone [9], Makinde *et al.* [10], Mehmoud and Ali [11] Makinde and Ibrahim [12] and Sheikholemi [13]; few among others.

Heat transfer flows with temperature-dependent thermal conductivity has been studied by investigators owing to its numerous applications in engineering technology as in petroleum industries for extraction of oil, extrusion of plastic sheets, polymer processing, spinning of fibers, cooling of elastic sheets etc. The quality of final products in manufacturing industries relies on the rate of heat transfer and therefore cooling procedure has to be controlled effectively. In heat sink/ source applications; materials of high thermal conductivity are widely used while those of low conductivity are used as insulators. For example, liquid metals having Prandtl number in the range 0.01 – 0.1 are generally used as coolants because of their high thermal conductivity. Correlated studies are in Van den Berg *et al.*[14], Van den Berg *et al.* [15] and Sharma and Rafi [16].

Studies related to flow of viscous fluids through channels with temperature-dependent viscousity are of great importance in industries such as in food processing, purification of underground oil, and coating of metals. Reynold [17] was first to proposed the early equation for temperature dependent viscosity where he stated that; the viscosity of liquid decreases exponentially with increase in temperature. Several other scholars have modeled the expression for temperature dependent fluid viscosity; all of which revolved around the ancient form of Reynold's expression and these can be viewed in [15], Elbashbashy *et al.* [18] and Mukhapdyay *et al.* [19]. Interrelated studies can be viewed in Grey *et al.* [20] where they submitted that; when varying viscosity property of fluid is included, the flow characteristics change substantially compared to the constant case while the studies of Macosco [21] disclosed that; in industrial systems fluid can be subjected to extreme conditions due to high temperature, pressure and shear rate and this affect its viscosity. Mehta and Sood [22] stressed that the usual assumption of constant viscosity property of fluids in the study of boundary layer flows is not enough to depict the true situation in the flow characteristics. Other correlated studies can be referred to Kafousius and Williams [23], Makinde and Ogulu [24] and Kafousius and Rees [25] where the latter concluded that when viscosity of a fluid is sensitive to temperature variation, the effect of temperature-dependent viscosity has to be taken into account otherwise considerable errors may occur in the heat transfer characteristics.

In all the above studies; the researchers discussed the effect of thermal radiation on the flow behavior using linearized Rosseland heat diffusion and this was however faulted by Magyari and Pantokratoras [26] on the basis that, it does not capture realistic behavior in the energy emission or conduction in boundary layer flows and they therefore proposed alternative approach using non-linearized temperature. In apprehension of this novel idea, connected studies can be seen in Mansour [27] and Jha *et al.* [28].

The present article investigates steady free convective flow through a vertical channel with variable fluid's properties and thermal radiation using Adomian decomposition method (Adomian [29]) and adopting non-linearized heat diffusion.

II. MATHEMATICAL FORMULATION

The physical problem under consideration consists of a vertical channel formed by two infinite parallel plate stationed h distance apart. The channel is filled with an optically dense viscous incompressible fluid in the presence of an incidence radiative heat flux of intensity q_r which is absorbed by the plates and transferred to the fluid as shown in Figure 1. Since the fluid is of an optically dense; the radiative heat flux of Rosseland heat diffusion is utilized to examine the energy equation in the flow formation. The x'-axis is taken along the channel in the vertically upward direction, being the direction of the flow while the y'-axis is taken normal to it with the effects of radiative heat flux in the x'- direction is considered negligible compared to that in the y'- direction. The temperature of the plate kept at $y' = 0$ raised or felt to T_w and there after remained constant while the other plate at $y' = h$ is fixed and maintained at temperature T_0.

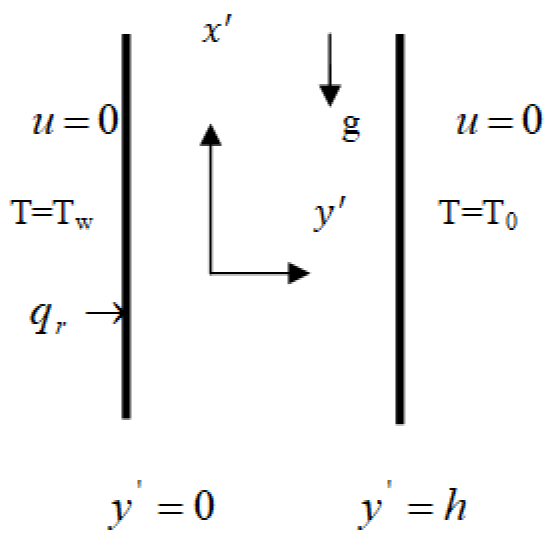

Figure 1. Schematic diagram of the problem

Under these assumptions; the appropriate governing equations are:

$$\frac{1}{\rho}\frac{\partial}{\partial y'}\left(\mu\frac{\partial u'}{\partial y'}\right)+g\beta\left(T'-T_0\right)=0 \qquad (1)$$

$$\frac{1}{\rho c_p}\frac{\partial}{\partial y'}\left(k\frac{\partial T'}{\partial y'}\right)-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y'}=0 \qquad (2)$$

with the initial and boundary conditions:

$$u^{'}=0, T^{'}=T_0 \quad for \quad 0\le y^{'}\le h \qquad (3)$$

and

$$\begin{cases} u^{'}=0 \quad T^{'}=T_w \quad at \quad y^{'}=0 \\ u^{'}=0, \quad T^{'}=T_0 \quad at \quad y^{'}=h \end{cases} \qquad (4)$$

The radiative heat flux (q_r)as given by Sparrow and Cess.[30] is utilized:

$$q_r=\frac{-4\sigma\,\partial T^{'4}}{3\,\delta\,\partial y^{'}} \qquad (5)$$

Following Carey and Mollendorf [31], the fluid viscosity and thermal conductivity assumed the forms:

$$\mu=\mu_0\left(1-\lambda\left(\frac{T-T_0}{T_w-T_0}\right)\right),$$

$$k=k_0\left(1-\varepsilon\left(\frac{T-T_0}{T_w-T_0}\right)\right) \quad for\ \lambda,\varepsilon\in\Re \qquad (6)$$

In order to transform equations (1) – (5) into dimensionless form; the following quantities are introduced:

$$u=\frac{u^{'}\upsilon}{g\beta\left(T^{'}-T_0\right)h^2},\ y=\frac{y^{'}}{h},\ \theta(y)=\frac{T^{'}-T_0}{T_w-T_0} \qquad (7)$$

Using equations (6) and (7) in equation (1); we get:

$$u^{'}(y)=\lambda\left(1+\lambda\theta(y)\right)\ \theta^{'}(y)\ u^{'}(y)-\theta(y) \qquad (8)$$

Following Magyari and Pantokratoras [26], $\dfrac{\partial q_r}{\partial y}$ is expanded as follow:

$$\frac{\partial q_r}{\partial y^{'}}=\frac{\partial}{\partial y^{'}}\left[\left(-\frac{4\sigma}{3\delta}\right)\frac{\partial T^{'4}}{\partial y^{'}}\right]=-\frac{4\sigma}{3\delta}\frac{\partial^2 T^{'4}}{\partial y^{'2}}$$

$$=-\frac{4\sigma}{3\delta h^2}\frac{\partial^2}{\partial y^{2}}\left(\left[\theta(y)(T_w-T_0)+T_0\right]^4\right)$$

$$=-\frac{4\sigma}{3\delta h^2}\frac{\partial}{\partial y}\left(\left(4\left[\theta(y)(T_w-T_0)+T_0\right]^3\right)\frac{\partial}{\partial y}\left(\theta(y)(T_w-T_0)+T_0\right)\right)$$

$$=-\frac{4\sigma}{h^2\delta}(T_w-T_0)^4\left(4\left[\theta(y)+\phi\right]^2\theta^{'}(y)\theta^{'}(y)+\frac{4}{3}\left[\theta(y)+\phi\right]^3\theta^{''}(y)\right) \qquad (9)$$

Substituting equation (9) in equation (2) and rearranging gives:

$$\theta^{''}(y)=\varepsilon\theta^{'2}(y)\left(1+\varepsilon\theta(y)\right)\left[1-\frac{4R_T}{3}\left(1+\varepsilon\theta(y)\right)\left[\theta(y)+\phi\right]^3\right]$$

$$-4R_T\left(1+\varepsilon\theta(y)\right)\left[\theta(y)+\phi\right]^2\theta^{'2}(y)\left[1-\frac{4R_T}{3}\left(1+\varepsilon\theta(y)\right)\left[\theta(y)+\phi\right]^3\right] \qquad (10)$$

Again, using equation (7) in equation (3) and (4), the initial and boundary conditions are now:

$$u=0, T=T_0 \quad for \quad 0\le y\le 1 \qquad (11)$$

$$\begin{cases} u=0, \quad \theta=1 \quad at \quad y=0 \\ u=0, \quad \theta=0 \quad at \quad y=1 \end{cases} \qquad (12)$$

Where

$$R_T=\frac{4\sigma(T_w-T_0)^3}{3k_0\delta},\ \phi=\frac{T_0}{T_w-T_0} \qquad (13)$$

III. ADM SOLUTION OF THE PROBLEM

The method of Adomian decomposition method which is popularly known as ADM was first introduced by Adomian [29] in an attempt to find solutions of differential equations in the form of series whose terms are calculated recursively using Adomian polynomials. Some of the benefits of this method over the known classical techniques include; it does not require discretization of the solution, the method avoids perturbation in order to find conditions required for next computations, the method does not result in any large system of equations neither does is it affected by computational round off errors and it does not require long time and large amount of computer memory.

Using ADM, the differential equations in equations (8) and (10) are written in the form:

$$Lu(y)=\lambda(1+\lambda\theta(y))\theta^{'}(y)u^{'}(y)-\theta(y) \qquad (14)$$

$$L\theta(y)=\varepsilon\theta'^{2}(y)\left(1+\varepsilon\theta(y)\right)\left[1-\frac{4R_T}{3}\left(1+\varepsilon\theta(y)\right)\left[\theta(y)+\phi\right]^3\right]$$

$$-4R_T\left(1+\varepsilon\theta(y)\right)\left[\theta(y)+\phi\right]^2\theta'^{2}(y)\left[1-\frac{4R_T}{3}\left(1+\varepsilon\theta(y)\right)\left[\theta(y)+\phi\right]^3\right] \quad (15)$$

where $Lu(y)=u''(y) \quad and \quad L\theta(y)=\theta''(y)$ (16)

Operating L^{-1} both sides of equations (14) and (15) we have:

$$L^{-1}Lu(y)=\lambda L^{-1}\left\{\left(1+\lambda\theta(y)\right)\theta'(y)u'(y)\right\}-L^{-1}\left\{\theta(y)\right\} \quad (17)$$

$$L^{-1}L\theta(y)=\varepsilon L^{-1}\left\{\theta'^{2}(y)\left(1+\varepsilon\theta(y)\right)\left[1-\frac{4R_T}{3}\left(1+\varepsilon\theta(y)\right)\left[\theta(y)+\phi\right]^3\right]\right\}$$

$$-4R_T L^{-1}\left\{\left(1+\varepsilon\theta(y)\right)\left[\theta(y)+\phi\right]^2\theta'^{2}(y)\left[1-\frac{4R_T}{3}\left(1+\varepsilon\theta(y)\right)\left[\theta(y)+\phi\right]^3\right]\right\} \quad (18)$$

Where $L^{-1}=\int\int(\bullet)dydy$ (19)

According to ADM:

$$\begin{cases} L^{-1}Lu(y)=u(y)-u(0)-yu'(0) \\ L^{-1}L\theta(y)=\theta(y)-\theta(0)-y\theta'(0) \end{cases} \quad (20)$$

Using equations (12) and (20) in equations (17) and (18) we get:

$$u(y)=yA+\lambda L^{-1}\left\{\left(1+\lambda\theta(y)\right)\theta'(y)u'(y)\right\}-L^{-1}\left\{\theta(y)\right\} \quad (21)$$

$$L^{-1}L\theta(y)=1+yB+L^{-1}\left\{\varepsilon\theta'^{2}(y)\left(1+\varepsilon\theta(y)\right)\left[1-\frac{4R_T}{3}\left(1+\varepsilon\theta(y)\right)\left[\theta(y)+\phi\right]^3\right]\right\}$$

$$-4R_T L^{-1}\left\{\left(1+\varepsilon\theta(y)\right)\left[\theta(y)+\phi\right]^2\theta'^{2}(y)\left[1-\frac{4R_T}{3}\left(1+\varepsilon\theta(y)\right)\left[\theta(y)+\phi\right]^3\right]\right\} \quad (22)$$

Where $A=f'(0)$ and $B=\theta'(0)$ are assumed values to be determined based on the boundary condition in equation (12).

According to standard ADM, $u(y)$ and $\theta(y)$ can be expressed in the forms:

$$u(y)=\sum_{n=0}^{\infty}u_n(y) \text{ and } \theta(y)=\sum_{n=0}^{\infty}\theta_n(y) \quad (23)$$

Using equation (23) in equations (22) and (22), we have:

$$u(y)=yA+\lambda L^{-1}\left\{\left(1+\lambda\sum_{n=0}^{\infty}\theta_n(y)\right)\frac{d}{dy}\left(\sum_{n=0}^{\infty}\theta_n(y)\right)\frac{d}{dy}\left(\sum_{n=0}^{\infty}u_n(y)\right)\right\}-L^{-1}\left\{\sum_{n=0}^{\infty}\theta_n(y)\right\} \quad (24)$$

$$\sum_{n=0}^{\infty}\theta_n(y)=1+yB+$$

$$\varepsilon L^{-1}\left\{\frac{d}{dy}\left(\sum_{n=0}^{\infty}\theta_n(y)\right)\frac{d}{dy}\left(\sum_{n=0}^{\infty}\theta_n(y)\right)\left(1+\varepsilon\sum_{n=0}^{\infty}\theta_n(y)\right)\left[1-\frac{4R_T}{3}\left(1+\varepsilon\sum_{n=0}^{\infty}\theta_n(y)\right)\left[\sum_{n=0}^{\infty}\theta_n(y)+\phi\right]^3\right]\right\}$$

$$-4R_T L^{-1}\left\{\left(1+\varepsilon\sum_{n=0}^{\infty}\theta_n(y)\right)\left[\sum_{n=0}^{\infty}\theta_n(y)+\phi\right]^2\frac{d}{dy}\left(\sum_{n=0}^{\infty}\theta_n(y)\right)\frac{d}{dy}\left(\sum_{n=0}^{\infty}\theta_n(y)\right)\left[1-\frac{4R_T}{3}\left(1+\varepsilon\sum_{n=0}^{\infty}\theta_n(y)\right)\left[\sum_{n=0}^{\infty}\theta_n(y)+\phi\right]^3\right]\right\} \quad (25)$$

Setting $\theta_0(y)=1+By$ and

$$u_0(y)=yA-L^{-1}\left\{\sum_{n=0}^{\infty}\theta_n(y)\right\} \quad (26)$$

then $u_{n+1}(y)$ and $\theta_{n+1}(y)$ for $n\geq 0$ are determined using the recursive relations:

$$u_{n+1}(y)=\lambda L^{-1}\left\{\left(1+\lambda\theta_n(y)\right)\frac{d}{dy}\left(\theta_n(y)\right)\frac{d}{dy}\left(u_n(y)\right)\right\} \quad (27)$$

$$\theta_{n+1}(y)=\varepsilon L^{-1}\left\{\frac{d}{dy}\left(\theta_n(y)\right)\frac{d}{dy}\left(\theta_n(y)\right)\left(1+\varepsilon\theta_n(y)\right)\left[1-\frac{4R_T}{3}\left(1+\varepsilon\theta_n(y)\right)\left[\theta_n(y)+\phi\right]^3\right]\right\}-$$

$$4R_T L^{-1}\left\{\left(1+\varepsilon\theta_n(y)\right)\left[\theta_n(y)+\phi\right]^2\frac{d}{dy}\left(\theta_n(y)\right)\frac{d}{dy}\left(\theta_n(y)\right)\left[1-\frac{4R_T}{3}\left(1+\varepsilon\theta_n(y)\right)\left[\theta_n(y)+\phi\right]^3\right]\right\} \quad (28)$$

For details on ADM refer to Adomian [29].

1) *Convergence of the ADM solution and termination criterion of the problem:*

It is well known that the convergence of ADM is rapidly convergent in Adomian [29] and Cherruault [32]. However; to confirm the convergence of the ADM solution in the present problem; the method of ratio test is deployed.

Using computer algebra package; the following ADM terms were obtained at $y = 0.5,\ \lambda = 0.1,\ \phi = 0.1,\ R_T = 0.1,\ \varepsilon = 0.1$ as:

$$\theta_0 = 0.534518207, \theta_1 = -0.02653683299, \theta_2 = 0.00002479802883,$$
$$\theta_3 = 1,7614491614*10^{-11}, u_0 = 0.06567502426, u_1 = -0.002539004359,$$
$$u_2 = 0.00005241076551, u_3 = -7.568784091*10^{-7} \tag{29}$$

Using ratio test for convergence and on using equation (30); the resulting numerical values are realized:

$$\left|\frac{\theta_1}{\theta_0}\right| = 0.04964626582, \left|\frac{\theta_2}{\theta_1}\right| = 0.0009344758223, \left|\frac{\theta_3}{\theta_2}\right| = 7.103353360*10^{-7}$$

$$\left|\frac{u_1}{u_0}\right| = 0.03866012, \left|\frac{u_2}{u_1}\right| = 0.0009344758223, \left|\frac{\theta_3}{\theta_2}\right| = 70.01443948 \tag{30}$$

The numerical values in equation (30) are observed to be contented with the ratio test formula for convergence; since $\lim_{j\to\infty}\left|\frac{f_{j+1}}{f_j}\right| < 1$, as $j \to \infty$ (Robert [33]). Hence the ADM solution of the present problem converges.

The series is truncated at a point such that the contribution of any additional term is negligible to the final solution. For this reason, the series is truncated whenever $|u_i, \theta_i| < \varepsilon$; where $\varepsilon = 3.5*10^{-4}$ is chosen. Bearing this assumption, the solution for u and θ are thus both terminated after the 3rd terms. The final solution is not displayed here due to its cumbersome size rather it is used for numerical computations and discussion of the result in the later sections.

Following Kay [34]; Nusselt number on the channel plates stationed at $y = 0$ and $y = 1$ are respectively evaluated using:

$$Nu_0 = (1 - \varepsilon\theta(y))\frac{d\theta}{dy}\Big|_{y=0} \text{ and}$$
$$Nu_1 = (1 - \varepsilon\theta(y))\frac{d\theta}{dy}\Big|_{y=1} \tag{31}$$

and the skin frictions on the plates are calculated via:

$$\tau_0 = (1 - \lambda\theta(y))\ \frac{du(y)}{dy}\Big|_{y=0} \text{ and}$$
$$\tau_1 = (1 - \lambda\theta(y))\ \frac{du(y)}{dy}\Big|_{y=1} \tag{32}$$

IV. RESULTS AND DISCUSSION

This section presents the simulated results of equations (26) – (28) in considering the influences of physical parameters involved and the results are presented graphically in figure 1 – 7 and on tables I - III. For the purpose of discussion of this result; the values of λ and ϕ are arbitrarily chosen between 0.1 – 3.0 while that of R_T is taken in the range $0 \le R_T \le 1$ for which the solution of the problem converges.

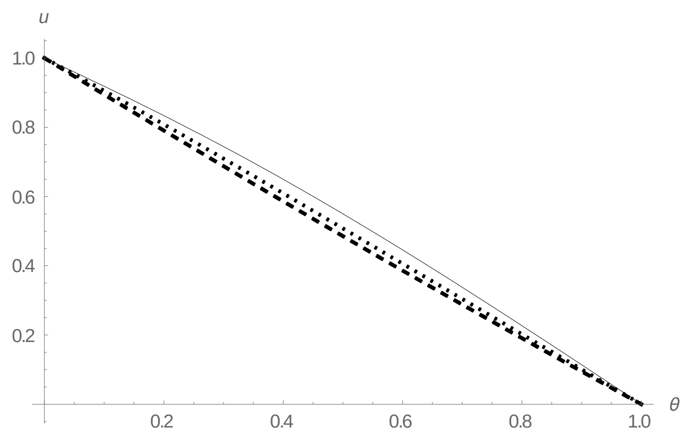

Fig 2: Temperature profiles for different R_T

($\phi = 0.1, \epsilon = 0.1$. – – – R_T= 0.001, …… R_T= 0.1, ______ R_T= 0.5)

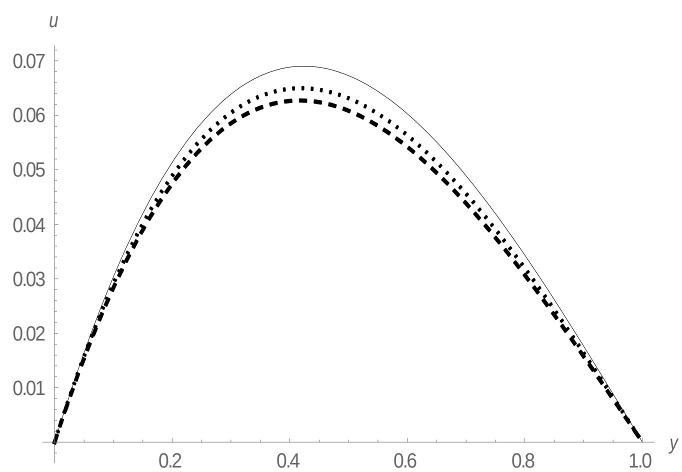

Fig 3: Velocity profiles for for different R_T

($\phi = 0.1, \lambda = 0.1,\ \epsilon = 0.1$,– – – R_T=0.001, …… R_T= 0.1,______ R_T= 0.5)

Figure 2 demonstrates the effect of thermal radiation on the fluid temperature within the channel where the fluid tempereture is seen to increase with increase in R_T. The consequential effect of this is reflected on figure 3where the fluid velocity also increases with increase in R_T, These behaviors are attributed to the decrease in thermal conduction of the working fluid.

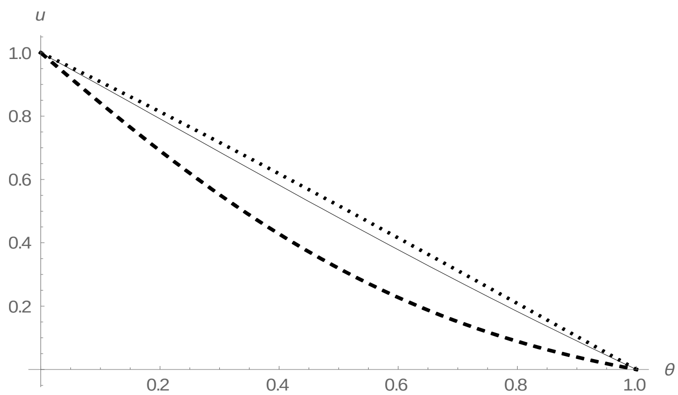

Fig 4: Temperature profiles for different ϵ:

($\phi = 0.1, R_T = 0.1$,……ϵ = 0.001,______ϵ =0.3, - - - - - - ϵ = 0.6)

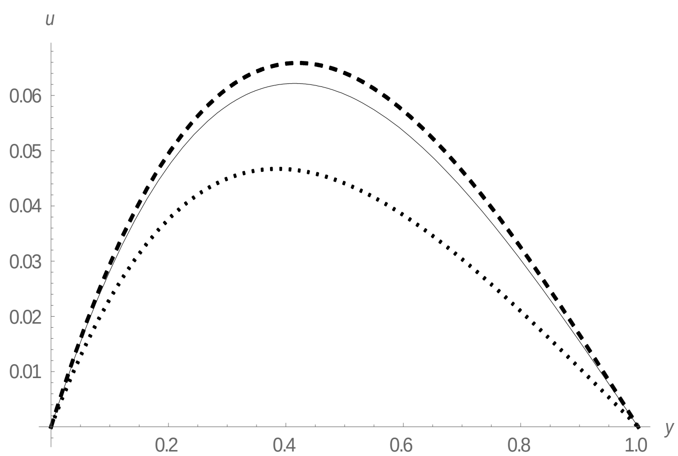

Fig 5: Velocity profile for different ϵ:

($\phi = 0.1, \lambda = 0.1, R_T = 0.1$, - - - - - ϵ =0.001,___ϵ =0.3,. . . . . . . ϵ = 0.6)

Figure 4 and 5 illustrate the effect of thermal conduction parameter (ϵ) on both the temperature and velocity of the fluid within the channel when other parameters were fixed. From these figures it is noticed that both the temperature and velocity of the fluid decreases with increase in ϵ. This physically reveals the effect of decrease in thermal diffusivity of the fluid with growing ϵ which act to diminish the influence of the applied boundary temperature which causes a decrease in the thermodynamics of the fluid.

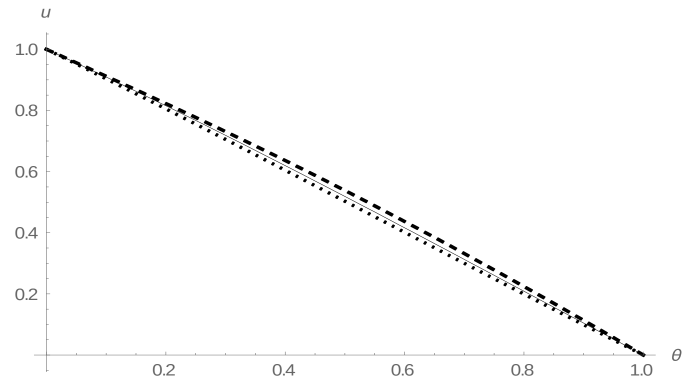

Fig 6: Temperature profiles for different ϕ:

($\epsilon = 0.1, R_T = 0.1, - - -\phi = 0.1$, ______$\phi = 0.2$, … ….$\phi = 0.4$)

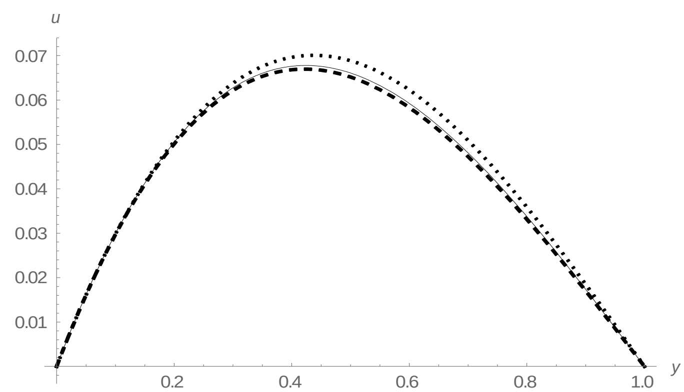

Fig 7: Velocity profiles for different ϕ:

($\lambda = 0.1$, $\epsilon = 0.1, R_T = 0.1, - - -\phi = 0.1$, ______$\phi = 0.2$, … ….$\phi = 0.4$)

Figure 6 displayed the influence of temperature difference parameter (ϕ) on the fluid temperature where it is seen that the temperture of the fluid within the channel increases with increase in ϕ. This trend is inclined to the increase in the ambient temperature of the working fluid and this resulted to the increase in the fluid velocity within the channel as graphed in figure 7.

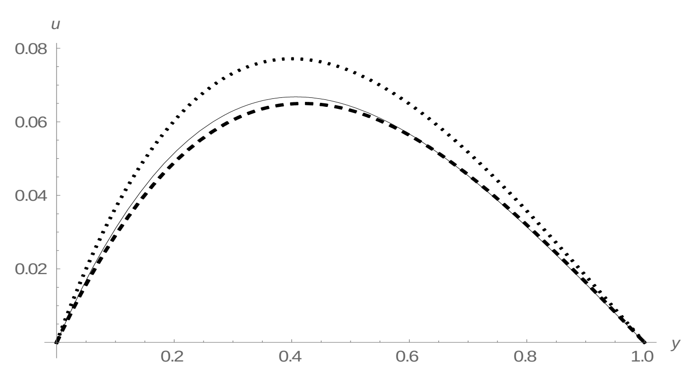

Fig 8: Velocity profile for different λ

$(\phi = 0.1,\ \epsilon = 0.1, R_T = 0.1, - - - \lambda = 0.1, \_\_\_\_\_ \lambda = 0.2$, -.-.- $\lambda = 0.6)$

Effect of varying viscosity variation parameter (λ) on the fluid velocity is exposed in figure 8 where the fluid velocity within the channel is viewed to increase with increase in λ. This behavior is owing to the decrease in the viscosity of the fluid.

Table I: Nusselt Number on the Channel Plates

R_T	$\varepsilon = 0.1$, $\phi = 0.1$		$\varepsilon = 0.4$, $\phi = 0.1$		$\varepsilon = 0.4$, $\phi = 0.4$	
	Nu_0	Nu_1	Nu_0	Nu_1	Nu_0	Nu_1
0.02	0.924437	0.95601	0.86404	0.59293	0.78436	0.66648
0.04	0.898543	0.96707	0.79169	0.66160	0.70389	0.76537
0.06	0.87589	0.98144	0.73910	0.71343	0.64771	0.83328
0.08	0.55747	0.97317	0.69870	0.75512	0.60978	0.88400
0.1	0.83787	1.00429	0.66657	0.78992	0.58323	0.92364

The effect of varying parameters on the rate of heat transfer (Nusselt number) between the working fluid and the channel plates is presented on Table I where Nu_0 is seen to decrease with increase in R_T. On the other hand Nu_1 is observed to increase with increase in R_T and later it decreases with further increase in R_T. For small increase in ε; Nu_0 decreases while Nu_1 decrease all with increase in R_T. Furthermore; with increase in ϕ; Nu_0 is observed to decreases while Nu_1 increases all with increase in R_T.

Table II: Numerical Values of Skin Friction on the Channel Plates

λ	$\varepsilon = 0.1$, $R_T = 0.01$, $\phi = 0.1$		$\varepsilon = 0.4$, $R_T = 0.01$, $\phi = 0.1$		$\varepsilon = 0.4$, $R_T = 0.04$, $\phi = 0.1$		$\varepsilon = 0.4$, $R_T = 0.04$, $\phi = 0.4$	
	τ_0	τ_1	τ_0	τ_1	τ_0	τ_1	τ_0	τ_1
0.1	0.30100	0.15894	0.26816	0.12453	0.27969	0.13572	0.28937	0.14555
0.3	0.24513	0.15405	0.22241	0.12401	0.23041	0.13397	0.23707	0.14254
0.5	0.18950	1.15673	0.17783	0.13222	0.18195	0.14064	0.18530	0.14759
0.7	0.13155	0.18044	0.13300	0.16725	0.13229	0.17219	0.31755	0.17573

Table II reflects the skin friction between the fluid and channel plates where τ_0 decreases with increase in λ and τ_1 decreases with initial increase in λ and later it increases with further increase in λ.Furthermore; with small increase in ε; both τ_0 and τ_1 decreaseswith increase in λ. Finally;the skin friction on the plates are observed to increase with increase in R_T and ϕ.

1) Validation of the result:

The present result on setting $R_T = \lambda = \varepsilon = 0$ is validated on comparison with the published work of Singh and Paul [1] on setting $\theta(1) = R = 0$ and it is tabulated below:

Table III: Comparison of the Present work with Published Study

y	Singh and Paul (2006) $\theta(1) = R = 0$		Present work $\lambda = R_T = \varepsilon = 0$	
	$\theta(y)$	$u(y)$	$\theta(y)$	$u(y)$
0.1	0.900	0.02850	0.900	0.02819
0.3	0.700	0.05950	0.700	0.05916
0.5	0.500	0.06250	0.500	0.06238
0.7	0.300	0.04550	0.300	0.04553
0.9	0.100	0.01650	0.100	0.01653

Numerical values in table III shows that the two studies are in good agreement with each other.

V. CONCLUSION

The present article investigated free convective flow through a vertical channel with variable fluid properties and thermal radiation using non-linear Rosseland heat diffusion, Adomian decomposition method and computer algebra package. The following major outcomes were deduced during the investigation:

i. Increase in thermal conduction parameter was found to decrease both the fluid velocity and temperature within the channel.

ii. Decrease in viscosity of the working fluid was recognized to increase the fluid velocity within the channel.

iii. Increase in thermal radiation parameter was realized to increase both the fluid's velocity and temperature in the channel.

ACKNOWLEDGEMENTS

The authors acknowledged the helping hand of Prof. B. K. Jha of the Department of Mathematics, Ahmadu Bello University, Zaria for tutoring and suggestive criticism during the write up. Also the authors acknowledge Federal University Dutsin-Ma for the release granted to the second author to undergo his postgraduate program.

Nomenclature and Greek symbols:

Symbols	*Interpretation*	*Unit*
y'	Dimensional length	m
y	Dimensionless length	
g	Acceleration due to gravity	ms^{-2}
k	Thermal conductivity	W/mK
T	Dimensional temperature	K
h	Dimensional channel width	m
T_w	Wall temperature	K
T_0	Ambient temperature	K
u'	Dimensional velocity	ms^{-1}
u	Dimensionless velocity	
υ	Kinematic viscosity of the fluid	m^2s^{-1}
α	Thermal diffusivity	m^2s^{-1}
δ	Absorption coefficient	
β	Volumetric expansion coefficient	K^{-1}
μ	Variable fluid viscosity	$kgm^{-1}s^{-1}$
μ_0	Dynamic fluid viscosity	$kgm^{-1}s^{-1}$
R_T	Thermal radiation parameter	
S	Heat generating/absorbing parameter	
q_r	Radiative heat flux	Wm^{-2}
ϕ	Temperature difference parameter	K
θ	Dimensionless temperature	
σ	Stefan-Boltzman constant	JK^{-1}
ε	Thermal conductivity variation parameter	
λ	Viscosity variation parameter	
$\Re$	Set of real numbers	
Nu_0	Nusselt number on the plate at $y = 0$	
Nu_1	Nusselt number on the plate at $y = 1$	
τ_0	Skin friction on the plate at $y = 0$	
τ_1	Skin friction on the plate at $y = 1$	

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