Predicting Momentum Effects of Fluid Flow in Reservoir Well Bore in Niger Delta Area

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***Abstract: -* The research on mass cum linear momentum balance model to predict fluid flow in a well bore of an oil reservoir is unveiled. The fundamental principles of mass cum momentum balance applied to control volume of material systems in this case a differential petroleum reservoir well bore. An intractable momentum balance model was derived from the principles of linear momentum. The model was solve using certified field data to generate values for profile plot of velocity and pressure parameters as a consequence of momentum of fluid flow in wellbore. The mass of the fluid flow in the well bore is a function of the pressure and the velocity sustenance in the well bore and this is an annexation of the momentum balance model. Finally, the pressure-velocity effects of reservoir well bore predicts sustainability of exponential production of crude oil from a given well bore.**

***Keywords:* Well-bore, oil reservoir, mass-cum-momentum-model, fluid-flow, pressure-velocity parameters**

I. INTRODUCTION

Rhetoric's apart, the research applies mass cum linear momentum balance model to fluid flow in a well bore of a reservoir in Niger Delta areas to predicting pressure-velocity parameters effects of fluid flow sustenance to the oil & gas facilities; an independent research carried out by [Wordu & Uzono, 2018] now published in reputable journal [Wordu & Uzono, 2019].

The flow of fluid from a reservoir well bore is a critical and difficult task in the oil and gas industry, especially for offshore operations. And, flow assurance is the act of delivering and assuring the transport of oil well stream fluid from the reservoir to the process facilities. Flow is therefore, design to identify, quantifies fluid mass and minimize the challenges with the flow risks such as: solids depositions to avoid reduction of well stream flow or clogging of the flow lines causing ceasing of production.

The main cause of ceasing of flow of fluid in well bore of any reservoir is waxes which are of diverse class of organic compounds [Archibong & Abam, 2019].

[Newberry, 1984] posited that all cases of organic deposits comprising paraffin and asphaltene are referred to as waxy crude oils; and Waxy crudes pose serious flow assurance issues in the oil industry particular in deep-water and frontier environments which are associated with very low temperatures and rapid pressure drops from long tieback lines that connect subsea wellheads to production facility. These crudes are difficult to handle because of their high pour-points leading to unplanned shutdowns as well as restart events due to the complex rheological properties of the gelled crude,

The Nigerian Niger Delta crude oil, which is the mainstay of Nigerian economy, exhibits waxiness, with deposits in the range of 30-45 % [Adewusi, 1997]; [Fasesan & Adewumi, 2003]; [Taiwo, 2009]; [Oladiipo, 2009]. Production tubing has also been known to wax up, necessitating frequent wax cutting, using scrapers conveyed by wire lines, which is an expensive practice. Billions of dollars have been lost to its prevention and remediation [Oladiipo, 2009]. The resultant effect on the petroleum industries include among others, reduced or deferred production, well shut-in, pipeline replacements and/or abandonment. For efficient operation of a pipeline system, steady and continuous flow without any interruption is desirable [Chang, 1999].

Waxy oils are found in west central Africa. Angola, Gabon, and Nigeria are examples of areas producing primarily waxy oils. Nigeria has a substantial reserve of paraffinic crude oils [Ajienka and Ikoku, 1997], known for their good quality (low sulphur, high API gravity), and containing moderate to high contents of paraffinic waxes. Characteristically, waxy crude oils have undesirably high pour points and are difficult to handle where the flowing and ambient temperatures are about or less than the pour-point. They exhibit non-Newtonian flow behaviour at temperature below the cloud point due to wax crystallization. Consequently, the pipeline transportation of petroleum crude oil from the production wells to the refineries is threatened. Petroleum wax consist mainly saturated paraffin hydrocarbons with number of carbon atoms in the range of 18–36. Wax may also contain small amounts of naphthenic hydrocarbons with their number of carbon atoms in the range of 30–60.

Finally, the studies focused on developing momentum model from principles of engineering research to predict the fluid flow in an ideal reservoir well bore that is unhindered by waxes of any kind and free of clogging materials of the well bore.

II. MATERIALS AND METHOD

2.1 Materials

The essential field properties and operational conditions are obtained from *reservoir well-bore in the Niger Delta.* Well-bores parameters are fluid flow properties and operational conditions given in table 1 below.

Table 1 Wellbore parameters, fluid properties and operational conditions

Vertical depth of the well, ft	**304.8m**	**1000**
Environmental geothermal gradient, $\frac{{}^{0}F}{ft}$	0.005	0.005
Tubing ID, ft	0.1m	0.329
Bottom hole flowing pressure, psia	10342.14kPa	1500
Temperature of the environment at pipe intake, ${}^{O}F$	121.1^{0}C	250
Temperature of the environment, ${}^{O}F$	82.22^{0}C	180
Mixture density, $\frac{lb_m}{ft^3}$	782.20kg/m^3	48.831
Gravitational force, $\frac{ftlb_m}{sec^2 lb_f}$	9.81m/s^2	9.8
Z-factor	0.87	0.87
Fluid flow rate, $\frac{lb_m}{sec}$	19087.07mol/s	42.08
Thermal conductivity of the earth, $\frac{Btu}{ft-\ {}^{0}F}$	17.64W/m.K	10.20
Overall heat transfer coefficient times pipe radius, $\frac{Btu}{hr-ft^2-\ {}^{0}F}$	0.908W/ m^{2}K	0.16

2.2 Method

Theoretical Constraints to the research

That, in engineering, dealing with liquids that are relatively incompressible the density

Rohr is essentially constant; and viscosity is reasonably constant, ie Dρ/Dt = 0.

Secondly, the fluid is Newtonian in character ie fluid follow Newtonian first law,, which means the viscosity is reasonably constant and not dependent on the rate of shear, The stresses τ_{xx}, τ_{yx}, τ_{zx} *can be related to the velocity gradients and viscosity,* μ.

.When this occurs, we obtain the Navier-Stokes equation for the x component, since Dv=0

2.2.1 Mass and Momentum Balance for 3-D Plane well-bore

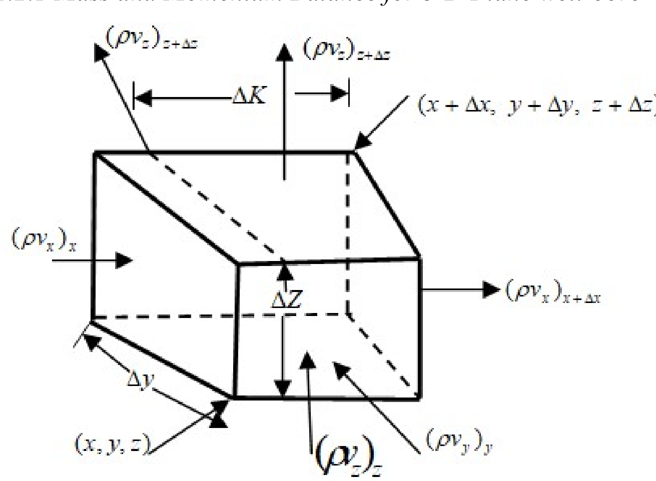

Figure 1 Mass and Momentum balance for elemental volume well-bore

$$\left\{\begin{matrix}\text{Rate of mass} \\ \text{in}\end{matrix}\right\} = \left\{\begin{matrix}\text{Rate of mass} \\ \text{out}\end{matrix}\right\} = \left\{\begin{matrix}\text{Rate of} \\ \text{Accumulation}\end{matrix}\right\} \quad \textbf{[1]}$$

Analyzing the m***ass balance*** equation [1] mathematically with proper notations on the *elemental volume* of reservoir well bore gives;

Rate of Accumulation = $\frac{\partial \rho}{\partial t}(\Delta x \Delta y \Delta z)$ of fluid *x, y, z* – direction **[2]**

Rate of mass in *x, y, z* direction =
$$\Delta y \Delta z((\rho v_x)_x) + \Delta x((\rho v_y)_y) + \Delta x \Delta y((\rho v_z)_z) \quad \textbf{[3]}$$

Rate of outflow of mass *x, y, z* direction =
$$\Delta y \Delta z((\rho v_x)_{x+\Delta x}) + \Delta x \Delta z((\rho v_y)_{y+\Delta y}) + \Delta x \Delta y((\rho v_z)z + \Delta z)$$
[4]

Combining equation (3.2) to (3.4) into equation [3.1] gives:

$$\frac{\partial \rho}{\partial t}\Delta x \Delta y \Delta z = \Delta y \Delta z[(\rho v_x)_x - (\rho v_x)_{x+\Delta x}] + \Delta x \Delta y[(\rho v_z)_z - (\rho v_z)_{z+\Delta z}] + \Delta x \Delta z[(\rho v_y)_y - (\rho v_y)_{y+\Delta d}]$$

$$\frac{\partial \rho}{\partial t}\Delta x \Delta y \Delta z = \Delta y \Delta z[(\rho v_x)_{x+\Delta x} - (\rho v_x)_x] - \Delta x \Delta y[(\rho v_z)_{z+\Delta z} - (\rho v_z)_z] + \Delta x \Delta z[(\rho v_y)_{y+\Delta z} - (\rho v_y)_y]$$
[5]

Dividing equation [5] by $\Delta x \; \Delta y \; \Delta z$ gives;

$$\frac{\partial \rho}{\partial t} = -\frac{\partial(\rho v_y)}{\partial x} - \frac{\partial(\rho v_z)}{\partial z}$$
[6]

But,

$$\frac{\partial \rho}{\partial t} = -\bar{v}(\rho v) \quad \textbf{[7]}$$

At steady state process, ***mass balance one-dimensional flow analysis,*** *x*–direction

$$\frac{\partial \rho}{\partial t} = 0;$$

$$\frac{\partial \rho}{\partial t} = -\frac{\partial(\rho v_x)}{\partial x} \quad \textbf{[8]}$$

$$0 = \frac{d(\rho v_x)}{\partial t} = \frac{d(\rho v)}{dL} = 0 \quad \textbf{[9]}$$

2.2.2 Momentum Model of Fluid Flow in Well-Bore Pipe

Taking a linear ***momentum balance of fluid flow in elemental reservoir well-bore*** figure [1] is stated in equation [10] as;

$$\left\{\begin{matrix}\text{Rate of momentum} \\ \text{Accumulated within} \\ \text{well-bore}\end{matrix}\right\} = \left\{\begin{matrix}\text{Rate of momentum} \\ \text{Entering well-bore} \\ \end{matrix}\right\} - \left\{\begin{matrix}\text{Rate of outflow of} \\ \text{momentum from} \\ \text{Well-bore}\end{matrix}\right\}$$

$$+ \left\{\begin{matrix}\text{Summation of forces} \\ \text{acting on the system} \\ \text{well-bore}\end{matrix}\right\} \quad \textbf{[10]}$$

Rate of accumulation = $\Delta x \ \Delta y \ \Delta z \ \frac{\partial}{\partial t}(\rho v_x)$ **[11]**

Rate of inflow of momentum =

$$\Delta x \ \Delta z\left((\rho v_x^2)_x\right) + \Delta x \ \Delta z\left(\rho(v_y \ v_y)_y\right) + \Delta x \ \Delta y\left((\rho v_z)_z\right)$$

Rate of outflow of momentum =

$$\Delta x \ \Delta z\left((\rho v_x . v_x)_{x+\Delta x}\right)$$
$$\Delta x \ \Delta z\left(\rho v_y \ v_y\right)_{y+\Delta y} + \quad \textbf{[12]}$$
$$\Delta x \ \Delta z\left(\rho vz \ vz\right)_{z+\Delta z}$$
$$+ \ \Delta x \ \Delta y\left((\rho v_z)_z\right)$$

Sum of forces acting on the well-bore system [***stress, pressure forces and the gravitational forces acting on the fluid]***

$$=\left(\tau_{x.x}\right)_x \ \Delta y \ \Delta x$$
$$+ \quad \left(p_x - p_{x+\Delta x}\right)\Delta y \ \Delta z \quad + \rho g x \ \sin \ \theta \ \Delta x \ \ \Delta y \ \Delta z$$

[13]

$$i.e \sum f_i = \left(\delta_x + p_x + F_y\right)$$

Combining these equations into equation [3.1] gives;

$$\Delta y \ \Delta x \frac{\partial(\rho v_x)}{\partial t} = -\Delta y \ \Delta z \ \partial(\rho v_x . v_x) - \Delta x \ \Delta y \ \partial(\rho v_z . v_z)$$

$$\Delta x \ \Delta z \ \partial(\rho v_y . v_y) - \Delta z \ \Delta y \ (\tau_{zx}) + \rho g \ \text{Sin} \ \theta$$
$$- \Delta y \ \Delta z \ (p_{x+\Delta x} - p_x)$$

[14]

Dividing all through by $\Delta x \ \Delta y \ \Delta z$ gives;

$$\frac{\partial(\rho v_x)}{\partial t} = -\left(\frac{\partial(\rho v_x v_x)}{\partial_x} + \frac{\partial(\rho v_z v_z)}{\partial_y} + \frac{\partial(\rho v_z v_z)}{\partial_z}\right)$$
$$- \left(\frac{\partial \tau_{xx}}{\partial_x} + \frac{\partial \tau_{yx}}{\partial_y} + \frac{\partial \tau_{zx}}{\partial_z}\right) - \frac{\partial p}{\partial_x} + \rho g \ \text{Sin} \ \theta$$

For one-dimensional plane alone:

$$\frac{\partial(\rho v_x)}{\partial t} = -\frac{\partial(\rho v_x v_x)}{\partial_x} - \frac{\partial \tau_{xx}}{\partial_x} - \frac{\partial p}{\partial_x} + \rho g \ \text{Sin} \ \theta$$

[15]

$$\frac{\partial(\rho v_x)}{\partial t} = -[\nabla.\rho v v] - [\nabla.\tau] - \nabla p + \rho g$$

[16]

At steady state process, momentum balance equation gives

$$0 = -\frac{\partial(\rho v_x^2)}{\partial_x} - \frac{\partial \tau}{\partial_x} - \frac{\partial p}{\partial_x} + \rho g \ \text{Sin} \ \theta$$

$$\Rightarrow \frac{d(\rho v^2)}{dL} = -{dP}/{dL} - \frac{d\tau}{dL} + \rho g \ \text{Sin} \ \theta$$

[17]

But,

$$d\tau/dL = \frac{\tau \pi d}{Ap}$$

[18]

Combining equations gives;

$$\frac{\partial(\rho v^2)}{dL} = -\frac{dP}{dL} - \frac{\tau \pi d}{Ap} + \rho g \ \text{Sin} \ \theta \quad \textbf{[19]}$$

Equation [3.19] mathematically resolves to;

$$\frac{d}{dL}(\rho v.v) = \frac{dP}{dL}\frac{\tau \pi d}{Ap} + \rho g \ \text{Sin} \ \theta$$

$$\rho v \frac{dv}{dL} + v\frac{d}{dL}(\rho v) = -\frac{\tau \pi d}{Ap} + \rho g \ \text{Sin} \ \theta \quad \textbf{[20]}$$

Substituting equation [9] steady state condition into equation [20] gives;

$$\rho v \frac{dv}{dL} + v\frac{dP}{dL} - \frac{\tau \pi d}{Ap} + \rho g \ \text{Sin} \ \theta$$
$$\Rightarrow \frac{dP}{dL} = -\rho v \frac{dv}{dL} - \frac{\tau \pi d}{Ap} + \rho g \ \text{Sin} \ \theta \quad \textbf{[21]}$$

Equation [21] delineates ***Momentum balance model for fluid flow in well-bore pipe at steady states conditions for resolution of the Pressure P [KPa] and Velocity V [m/s] of fluid flow in wellbore.***

2.3 Pressure drop analysis

Equation [21] evolves *pressure and velocity as two major variables for evaluation of the momentum* transfer differential equation. Pressure drop in well bore is a function of [Re] numbers range of 2000 – 4000, expressing laminar and turbulence flows in the reservoir well bore is applied. With the pressure equation it becomes easier to evaluate the velocity component of the momentum transfer differential model equation [21].

IV. RESULTS AND DISCUSSION

The model prediction on the two parameters of the momentum balance shows perfect trajectories of fluid flow in the well-bore at a steady state process highly sustained without encumbrances.

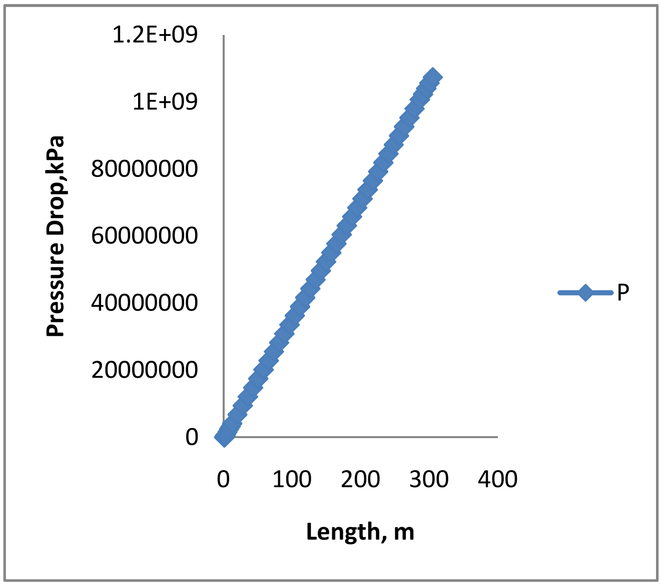

Figure 1 Pressure drop, kPa against depth (m)

The profile of pressure drop with length of the pipe is a linear function . As the length of the pipe increases, the greater the pressure drop.

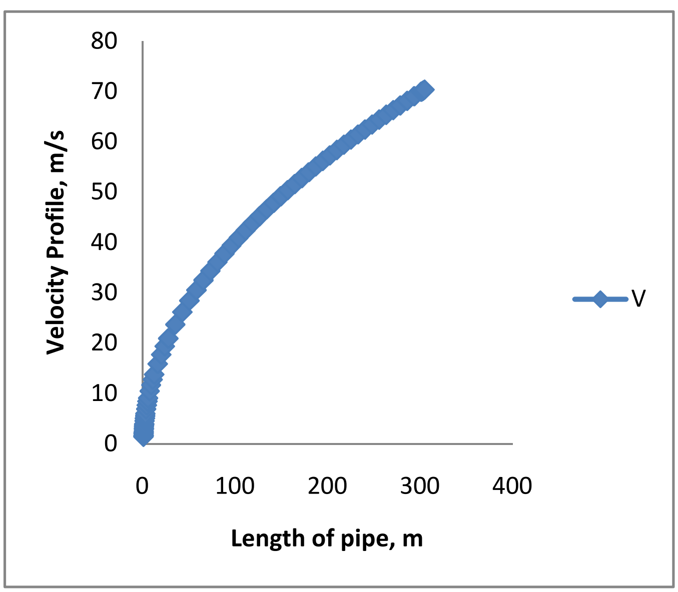

Figure 2 Velocity against Length [m]

Figure 2 velocity profile along the length of the pipe shows a exponential behaviour, mpiying increase in velocity from initial point to higher values of 70m as length increases from zero 300m.

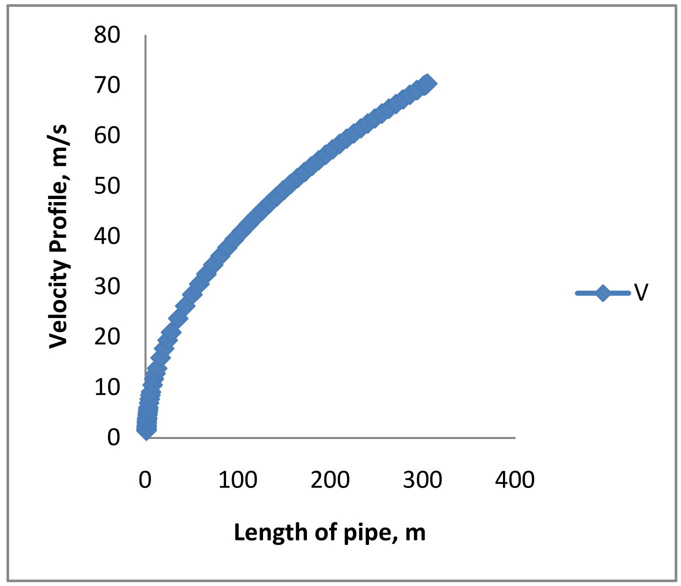

Figure 2 Velocity against Length [m]

The velocity along the length of the pipe shows a progressional behaviour, impiying increase in velocity from initial point to a maximum value in figure 2 more of well bore stream is sustaned to the facilities.

V. CONCLUSION

Ideally, the model predicted adequately exponential fluid flow in well bore of a reservoir which is unhindered of waxes of all kinds and free of clogging in the well bore.

NOMENCLATURE

ρ = density of the fluid, kg/m^3

v = velocity of the fluid, m/s

P = pressure in N/mm^2 or Pa.

L = length, m

τ = Torque, Pa (N/m^2)

d = Well-bore pipe diameter, m

A_p = Cross-Sectional area, m^2

θ = Angle of inclination, horizontal to the plane, in ^{0}Celsius or Kelvin

g = gravitational acceleration, m/s^2

REFERENCES

[1]. Archibong E. E & Abam, T. K, [2019] Predicting Wax formation temperature, Master's degree, Dissertation, Petroleum Engineering research, Rivers State University, Port Harcourt, Nigeria.

[2]. Adewusi, V. A [1997] Prediction of wax deposition potential of hydrocarbon systems for viscosity-pressure correlations. *Fuel* 76:1079–1083. Crude Oil Exploration in the World 152.

[3]. Ajienka, J. A [1983] The effect of Temperature on rheology of Waxy crude oils and its implication in production operation, Ph.D Dissertation, University of Port Harcourt, Nigeria

[4]. Ajienka, J. A & Ikoku, C.V, [1997] Waxy crude oil handling in Nigeria: practices, problems and prospects, *Energy sources*, Vol. 12. No.4. pp. 463-478

[5]. Chang C.; Nguyen Q. D. & Rønningsen H. P. [1999] Isothermal start-up of pipeline transporting waxy crude oil. *Journal of Non-Newtonian Fluid Mechanics*. 87 (1999) 127–154

[6]. Chang, C.; Boger, D. V. & Nguyen Q. D. [2000] Influence of thermal history on the waxy structure of statically cooled waxy crude oil. *SPE Journal*. (2000) Vol. 5:148-157.

[7]. Fasesan, S. O., and Adewunmi, O. O. [2003] Wax variation in some Nigerian oil wells in Delta Field. *Petroleum Science and Technology*, 21:91–111. Crude Oil Transportation: Nigerian Niger Delta Waxy Crude 153

[8]. Newberry, M. E. [1984] Chemical Effects on Crude Oil Pipeline Problems, Journal of Pet. Tech. 36(05) 779-786.

[9]. Oladiipo A., Bankole A. and Taiwo E [2009] Artificial Neural Network Modeling of Viscosity and Wax Deposition Potential of Nigerian Crude Oil and Gas Crude Oil Exploration in the World 154 Condensates. *33rd Annual SPE International Technical Conference and Exhibition in Abuja, Nigeria,* August 3-5, 2009. SPE 128600

[10]. Taiwo, E. A., Fasesan, S. O. and Akinyemi, O. P. [2009] Rheology of Doped Nigerian Niger-Delta Waxy Crude Oil. *Petroleum Science and Technology*, 27(13),1381— 1393

[11]. Wordu, A. A & Uzono, R. I, [2019] Predicting linear momentum effects of fluid flow in reservoir well bore