Mathematical Model for the Prediction of the Compressive Strength Characteristics of Concrete Made With Sugar Cane Bagasse Ash (SCBA)

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Abstract: **- This research examined the use of sugarcane bagasse ash (SCBA) as a supplementary material in concrete. The coarse aggregates were granite chippings from Abakaliki, Nigeria and fine aggregates from Amansea River, also in Nigeria. The sugarcane bagasse was from a Sugar Processing Plant, also in Nigeria. The aggregates were tested for physical and mechanical properties based on BS 812: Part 2 & Part 3:1975. A total number of 90 cubes were made, cured and tested according to BS 1881: Part 108; BS 1881: Part 111 & BS 1881: Part 116. Scheffe's (5, 2) lattice polynomial was used to develop a mathematical model for the optimization of the compressive strength of the sugarcane bagasse concrete at 28th day. The mathematical model developed was $\hat{Y}$ = 22.13 $X_1(2X_1\text{-}1)$ + 29.08 $X_2(2X_2\text{-}1)$+ 22.52 $X_3(2X_3\text{-}1)$+ 15.59 $X_4(2X_4\text{-}1)$+ 15.15$X_5(2X_5\text{-}1)$ + 98.68 $X_1 X_2$ + 83.76 $X_1 X_3$ +72.64$X_1 X_4$ + 91.44 $X_1 X_5$ + 97.68 $X_2 X_3$ +54.68X_2X_4+57.12X_2X_5+ 68.1668X_3X_4 + 80.04X_3X_5 + 58.88X_4X_5. The student's t-test and the Fisher test were used to test the adequacy of this model. The strengths predicted by the model were in complete agreement with the experimentally obtained values and the null hypothesis was satisfied.**

Keywords: **sugarcane bagasse, aggregate, model, Fisher's test, granite**

I. INTRODUCTION

Concrete is composed mainly of four materials, namely, cement, fine aggregate, coarse aggregate and water, and an additional material, known as an admixture, is sometimes added to modify certain of its properties [1]. Cement is the chemically active constituent but its reactivity is only brought into effect on mixing with water. The aggregate plays no part in chemical reactions but its usefulness arises because it is an economical filler material with good resistance to volume changes which take place within the concrete after mixing, and it improves the durability of the concrete [2]. It is important that concrete should have certain specified properties, and it is to be produced as economically as possible — a basic requirement in engineering [3]. Hence, there is need to optimize concrete properties such as strength [4]. Optimization is the determination of the optimal (maximum or minimum) value of a given function called the objective function, subject to a set of stated restrictions, or constraints, placed on the variables concerned [5]. This work optimized the use of SCBA as a supplementary material in concrete.

A. The Scheffe's (5, 2) Lattice Polynomial

Simplex is the structural representation of the line or planes joining the assumed positions of the constituent materials (atoms) of a mixture [1]. Scheffe [6] considered experiments with mixtures of which the property studied depended on the proportions of the components present but not on the quantity of the mixture [4]. If a mixture has a total of q components and x_i be the proportion of the ith component in the mixture such that $X_i \geq 0$ (i = 1, 2… q), then

$$X_1+ X_2+X_3 +\dots\dots\dots\dots\dots+ X_q = 1 \qquad (1)$$

For a 5-component mixture, Equation (1) becomes

$$X_1+ X_2+X_3 + X_4 + X_5 = 1 \qquad (2)$$

Scheffe [6] described mixture properties by reduced polynomials obtainable from Equation (3):

$$\hat{Y} =b_0+\Sigma b_iX_i+\Sigma b_{ij} X_i X_j+\Sigma b_{ijk} X_i X_j X_k +\dots\dots\dots+ e \qquad (3)$$

Where b_i , b_{ij} and b_{ijk} are constant coefficients; X_i , X_j and X_k are the pseudo components; and e is the random error term. Substituting the values of *i* and *j* will give

$$\begin{aligned}\hat{Y} &= b_0 + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_5X_5 \\ &+ b_{12}X_1X_2 + b_{13}X_1X_3 + b_{14}X_1X_4 + b_{15}X_1X_5 + \\ &b_{23}X_2X_3 + b_{24}X_2X_4 + b_{25}X_2X_5 + b_{34}X_3X_4 + \\ &b_{35}X_3X_5 + b_{45}X_4X_5 + b_{11}X_1{}^2 + b_{22}X_2{}^2 + b_{33}X_3{}^2 \\ &+ b_{44}X_4{}^2 + b_{55}X_5{}^2 + e \qquad (4)\end{aligned}$$

Multiplying Eqn.(2) by b_0 and multiplying the outcome by X_1, X_2, X_3, X_4 and X_5 in turn and substituting into Eqn. (4), we have:

$\hat{Y} = b_0 X_1+b_0 X_2+ b_0 X_3 + b_0 X_4+ b_0 X_5+ b_1 X_1 + b_2 X_2+ b_3X_3+ b_4X_4 + b_5X_5 + b_{12} X_1 X_2+ b_{13} X_1 X_3+b_{14} X_1 X_4+ b_{15} X_1 X_5 + b_{23}X_2X_3+ b_{24}X_2X_4+ b_{25} X_2 X_5 + b_{34}X_3X_4+ b_{35} X_3 X_5+ b_{45} X_4 X_5+ b_{11}(X_1\text{-} X_1X_2 - X_1X_3 - X_1X_4+ X_1X_5) + b_{22}(X_2\text{-} X_1X_2 - X_2X_3 -$

$X_2X_4 + X_2X_5) + b_{33}(X_3 - X_1X_3 - X_2X_3 - X_3X_4 + X_3X_5) + b_{44}(X_4 - X_1X_4 - X_2X_4 - X_3X_4 + X_4X_5) + b_{55}(X_5 - X_1X_5 - X_2X_5 - X_3X_5 + X_4X_5) + e$ (5)

Re-arranging Eqn. (5), we have

$$\hat{Y} = \Sigma \propto_i X_i + \Sigma \propto_{ij} X_i X_j \quad (6)$$

Where $1 \le i \le q$, $1 \le i \le j \le q$, $1 \le i \le j \le q$ respectively and

$\propto_i = b_0 + b_i + b_{ii}$ and $\propto_{ij} = b_{ij} + b_{ii} + b_{ii}$ (7)

Let the response function to the pure components (x_i) be denoted by y_i and the response to a 1:1 binary mixture of components i and j be y_{ij}. From Eqn. 6, it can be written that

$$\Sigma \propto_i X_i = \Sigma y_i X_i \quad (8)$$

Where (i = 1 to 5)

Evaluating y_i, for instance gives:

$$y_i = \propto_i \quad (9)$$

Also evaluating y_{ij}, gives in general the equations of the form

$$\propto_{ij} = 4y_{ij} - 2y_i - 2y_j \quad (10)$$

For the (5, 2) lattice polynomial, that is Eqn. 6 becomes:

$$\hat{Y} = y_1 X_1 + y_2 X_2 + y_3 X_3 + y_4 X_4 + y_5 X_5 + (4y_{12} - 2y_1 - 2y_2) X_1 X_2 + (4y_{13} - 2y_1 - 2y_3) X_1 X_3 + (4y_{14} - 2y_1 - 2y_4) X_1 X_4 + (4y_{15} - 2y_1 - 2y_5) X_1 X_5 + (4y_{23} - 2y_2 - 2y_3) X_2 X_3 + (4y_{24} - 2y_2 - 2y_4) X_2 X_4 + (4y_{25} - 2y_2 - 2y_5) X_2 X_5 + (4y_{34} - 2y_3 - 2y_4) X_3 X_4 + (4y_{35} - 2y_3 - 2y_5) X_3 X_5 + (4y_{45} - 2y_4 - 2y_5) X_4 X_5 + e \quad (11)$$

Expanding and factorizing Eqn.(11) gives

$$\begin{aligned} \hat{Y} &= y_1X_1(1-2X_2-2X_3-2X_4-2X_5)+ \\ &y_2X_2(1-2X_1-2X_3-2X_4-2X_5)+ \\ &y_3X_3(1-2X_1-2X_2-2X_4-2X_5) \\ &+y_4X_4(1-2X_1-2X_2-2X_3-2X_5)+ \\ &y_5X_5(1-2X_1-2X_2-2X_3-2X_4)+ \\ &4y_{12}X_1X_2+4y_{13}X_1X_3+4y_{14}X_1X_4+ \\ &4y_{15}X_1X_5+4y_{23}X_2X_3+4y_{24}X_2X_4+ \\ &4y_{25}X_2X_5+4y_{26}X_2X_6+4y_{34}X_3X_4+ \\ &4y_{35}X_3X_5+4y_{45}X_4X_5+e \end{aligned} \quad (12)$$

Multiplying Eqn. (2) by 2 and subtracting 1 from both sides of it gives

$$2X_1 - 1 = 1 - 2X_2 - 2X_3 - 2X_4 - 2X_5 \quad (13)$$

Similarly,

$$\left.\begin{aligned} 2X_2 - 1 &= 1 - 2X_1 - 2X_3 - 2X_4 - 2X_5 \\ 2X_3 - 1 &= 1 - 2X_1 - 2X_2 - 2X_4 - 2X_5 \\ 2X_4 - 1 &= 1 - 2X_1 - 2X_3 - 2X_4 - 2X_5 \\ 2X_5 - 1 &= 1 - 2X_1 - 2X_2 - 2X_3 - 2X_4 \end{aligned}\right\} \quad (14)$$

Substituting Eqns.(13) & (14) into Eqn.(12) gives

$$\begin{aligned} \hat{Y} &= y_1X_1(2X_1-1)+y_2X_2(2X_2-1)+y_3X_3(2X_3-1) \\ &+y_4X_4(2X_4-1)+y_5X_5(2X_5-1) \\ &+4y_{12}X_1X_2+4y_{13}X_1X_3+ \\ &4y_{14}X_1X_4+4y_{15}X_1X_5 \\ &+4y_{23}X_2X_3+4y_{24}X_2X_4+4y_{25}X_2X_5 \\ &+4y_{34}X_3X_4+4y_{35}X_3X_5+4y_{45}X_4X_5+e \end{aligned} \quad (15)$$

Eqn. (15) is the model for the optimization of a 5-component concrete mixture.

B. The Unbiased Estimate of the Unknown Variance

The unbiased estimate of the unknown variance S^2 is given by Biyi [7],

$$S_y = \frac{\sum (y_i - y)^2}{n-1} \quad (16)$$

If $a_i = X_i(2X_i - 1)$, $a_{ij} = 4X_iX_j$; for ($1 \le i \le q$) and ($1 \le i \le j \le q$) respectively.

Then, $\varepsilon = \Sigma a_i^2 + \Sigma a_{ij}^2$ (17)

where ε is the error of the predicted values of the response.

The t-test statistic is given by [7]

$$t = \left(\frac{\Delta y \sqrt{n}}{S_y}\right)\sqrt{(1+\varepsilon)} \quad (18)$$

Where $\Delta y = y_0 - y_t$; y_0 = observed value, y_t = theoretical value; n = number of replicate observations at every point; ε = as defined in Eqn.(17).

C. The Fisher's Test

The Fishers-test statistic is given by

$$F = \frac{S_1^2}{S_2^2} \quad (19)$$

The values of S_1 (lower value) and S_2 (upper value) are calculated from Eqn. (16).

II. MATERIALS AND METHOD

A. Preparation, Curing and Testing Of Cube Samples

The aggregates were sampled in accordance with the methods prescribed in BS 812: Part 1:1975 [8]. The test sieves were selected according to BS 410:1986 [9]. The water absorption, the apparent specific gravity and the bulk density of the coarse aggregates were determined following the procedures prescribed in BS 812: Part 2: 1975 [10]. The Los Angeles abrasion test was carried out in accordance with ASTM. Standard C131: 1976 [11]. The sieve analyses of the fine and coarse aggregate samples satisfied BS 882:1992 [12]. The sieving was performed by a sieve shaker. The water used in preparing the experimental samples satisfied the conditions prescribed in BS 3148:1980 [13]. The required concrete specimens were made in threes in accordance with the method specified in BS 1881: 108:1983 [14]. These specimens were cured for 28 days in accordance with BS 1881: Part 111: 1983 [15]. The testing was done in accordance with BS 1881: Part 116:1983 [16] using compressive testing machine.

B. Testing the Fit of the Quadratic Polynomials

The polynomial regression equation developed was tested to see if the model agreed with the actual experimental results. The null hypothesis was denoted by H_0 and the alternative by H_1.

Table 1: Design matrix for trial mixes based on Scheffe's (5, 2) Lattice Polynomial

	Values of Actual Components						Values of Pseudo Components				
S/No	S_1	S_2	S_3	S_4	S_5	Responses	X_1	X_2	X_3	X_4	X_5
1	0.60	0.95	0.05	1.5	2	22.13	1	0	0	0	0
2	0.50	0.90	0.10	1	2	29.08	0	1	0	0	0
3	0.55	0.85	0.15	2	5	22.52	0	0	1	0	0
4	0.65	0.80	0.20	3	6	15.59	0	0	0	1	0
5	0.57	0.75	0.25	2.4	3.6	15.15	0	0	0	0	1
6	0.55	0.925	0.075	1.25	2	24.67	0.5	0.5	0	0	0
7	0.575	0.90	0.10	1.75	3.5	20.94	0.5	0	0.5	0	0
8	0.625	0.875	0.125	2.25	4	18.16	0.5	0	0	0.5	0
9	0.585	0.85	0.15	1.95	2.8	22.86	0.5	0	0	0	0.5
10	0.525	0.875	0.125	1.5	3.5	24.42	0	0.5	0.5	0	0
11	0.575	0.85	0.15	2	4	13.67	0	0.5	0	0.5	0
12	0.585	0.825	0.175	1.7	2.8	14.28	0	0.5	0	0	0.5
13	0.6	0.825	0.175	2.5	5.5	17.04	0	0	0.5	0.5	0
14	0.56	0.80	0.20	2.2	4.3	20.01	0	0	0.5	0	0.5
15	0.61	0.775	0.225	2.7	4.8	14.72	0	0	0	0.5	0.5

LEGEND: Z_1= water; Z_2=Cement; Z_3 = SCBA, Z_4=Fine aggregate; Z_5=Coarse aggregate

Table 2: Design matrix for Control mixes based on Scheffe's (5, 2) Lattice Polynomial

	Values of Actual Components						Values of Pseudo Components				
S/No	S_1	S_2	S_3	S_4	S_5	Responses	X_1	X_2	X_3	X_4	X_5
1	0.55	0.90	0.10	1.5	3	22.86	¼	¼	¼	¼	0
2	0.6	0.866	0.133	2.167	4.333	18.28	¼	¼	¼	0	¼
3	0.607	0.833	0.167	2.3	3.867	19.29	¼	¼	0	¼	¼
4	0.575	0.875	0.125	1.875	3.75	18.78	¼	0	¼	¼	¼
5	0.592	0.838	0.163	2.225	4.15	19.60	0	¼	¼	¼	¼
6	0.579	0.870	0.130	1.941	3.808	17.67	$^1/_8$	¼	¼	¼	$^1/_8$
7	0.587	0.859	0.141	2.048	3.838	21.38	$^1/_8$	¼	¼	$^1/_8$	¼
8	0.593	0.853	0.147	2.141	4.025	14.41	$^1/_8$	¼	$^1/_8$	¼	¼
9	0.576	0.875	0.125	1.868	3.59	19.21	$^1/_8$	$^1/_8$	¼	¼	¼
10	0.58	0.868	0.133	1.938	3.67	22.20	$^1/_8$	$^1/_8$	$^1/_8$	$^1/_8$	½
11	0.463	0.696	0.104	1.553	3.047	20.38	½	$^1/_8$	$^1/_8$	$^1/_8$	$^1/_8$
12	0.574	0.869	0.131	1.88	3.553	20.44	$^1/_8$	½	$^1/_8$	$^1/_8$	$^1/_8$
13	0.59	0.903	0.131	1.9	3.7	21.38	$^1/_8$	$^1/_8$	½	$^1/_8$	$^1/_8$
14	0.59	0.901	0.133	1.907	3.677	21.13	$^1/_8$	$^1/_8$	$^1/_8$	½	$^1/_8$
15	0.589	0.912	0.122	1.848	3.66	18.42	$^1/_8$	$^1/_8$	$^1/_8$	$^1/_8$	½

LEGEND: Z_1= water; Z_2=Cement; Z_3 = SCBA, Z_4=Fine aggregate; Z_5=Coarse aggregate

III. RESULTS AND DISCUSSION

A. Physical and Mechanical Properties of Aggregates

Sieve analyses of both the fine and coarse aggregates were performed and the grading curves shown in Figures 1 and 2 respectively. These grading curves showed the particle size distribution of the aggregates. The maximum aggregate size for the granite chipping was 20 mm and 2mm for the fine sand. The granite chippings had water absorption of 2.7%, moisture content of 44.2%, apparent specific gravity of 2.26, Los Angeles abrasion value of 22% and bulk density of 2072.4 kg/m^3.

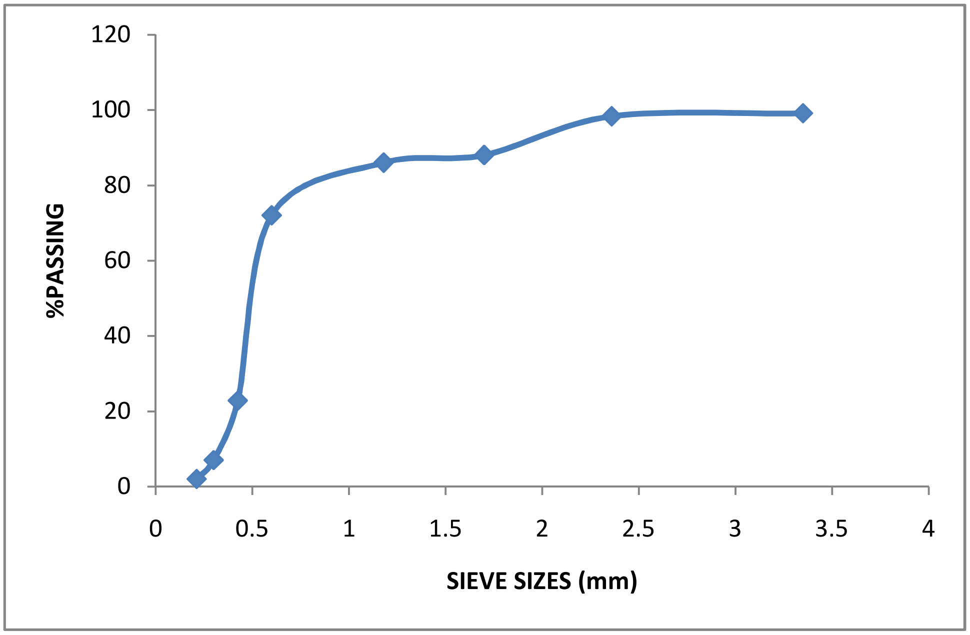

Figure 1 Grading Curve of Sharp Sand

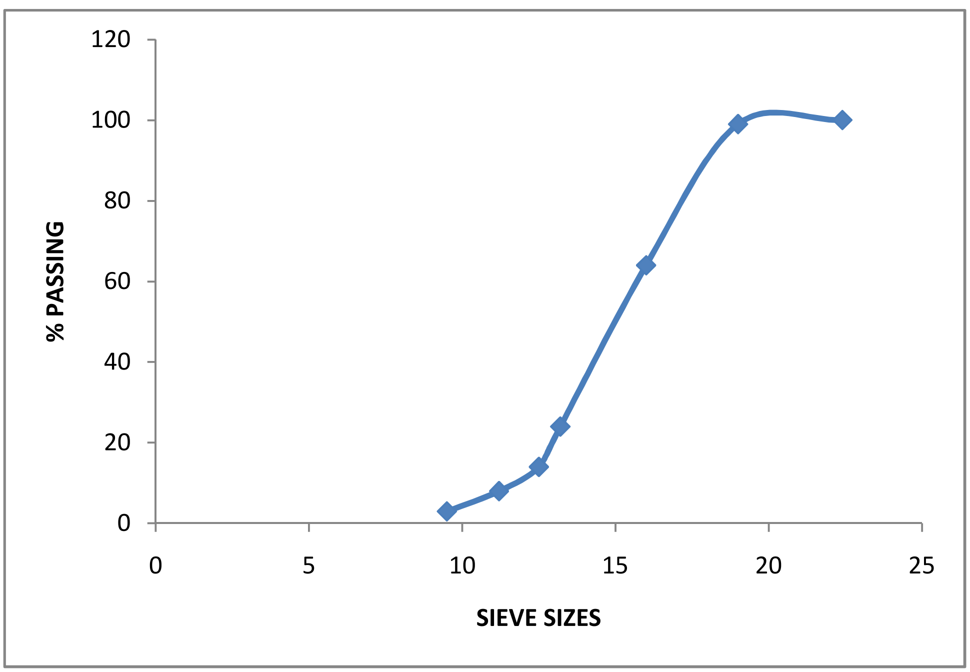

Figure 2 Grading Curve of Coarse aggregate

B. The Regression Equation for the Compressive Strength Tests Results

Applying the responses (average compressive strengths) of Tables 1 & 2 to Eqn. (15) gives:

$$\hat{Y} = 22.13X_1(2X_1 - 1) + 29.08X_2(2X_2 - 1) + 22.52X_3(2X_3 - 1) + 15.59X_4(2X_4 - 1) + 15.15X_5(2X_5 - 1) + 98.68X_1X_2 + 83.76X_1X_3 + 72.64X_1X_4 + 91.44X_1X_5 + 97.68X_2X_3 + 54.68X_2X_4 + 57.12X_2X_5 + 68.16X_3X_4 + 80.04X_3X_5 + 58.88X_4X_5 + e \quad (20)$$

Eqn. (20) is the model for the optimization of the compressive strength of the sugar cane bagasse ash concrete at 28th day strength, based on Scheffe's (5, 2) polynomial.

C. Fit of the Polynomial

The scope of the work was represented as the design matrices for trial and control mixes based on Scheffe's (5, 2) lattice polynomial (Tables 1 & 2). The polynomial regression equation developed was tested to see if the model agreed with the actual experimental results. There was no significant difference between the experimental and the theoretically expected results. The null hypothesis, H_0 was satisfied.

D. F-Statistic Analysis

The sample variances S_1^2 and S_2^2 for the two sets of data were not significantly different (Table 3). It implied that the error(s) from experimental procedure were similar and that the sample variances being tested are estimates of the same population variance. Based on Eqn.(16), we had that $S_K^2 = 62.16825/14 = 4.4406$, $S_E^2 = 59.0435/14 = 4.2174$ & $F = 4.4406/4.2174 = 1.053$. From Fisher's table, $F_{0.05(14,14)} = 2.349$, hence the regression equation for the compressive strength of the SCBA concrete was adequate.

Table 3 F –Statistic For The Controlled Points, SCBA Concrete Compressive Strength, Based On Scheffe's (5, 2) Polynomial

Response symbol	y_K	y_E	$y_K - \breve{y}_K$	$y_E - \breve{y}_E$	$(y_K - \breve{y}_K)^2$	$(y_E - \breve{y}_E)^2$
C_1	22.86	23.125	3.17	3.10	10.018	9.626
C_2	18.28	18.299	-1.41	-1.72	1.992	2.970
C_3	19.29	19.466	-0.40	-0.56	0.161	0.310
C_4	18.78	18.725	-0.91	-1.30	0.831	1.683
C_5	19.60	19.474	-0.09	-0.55	0.008	0.301
C_6	17.62	17.999	-2.07	-2.02	4.305	4.094
C_7	21.38	21.53	1.69	1.51	2.851	2.273
C_8	14.41	15.617	-5.28	-4.41	27.895	19.408
C_9	19.21	19.408	-0.48	-0.61	0.229	0.377
C_{10}	22.20	22.29	2.51	2.27	6.309	5.142
C_{11}	20.38	22.308	0.69	2.29	0.469	5.224
C_{12}	20.44	20.559	0.75	0.54	0.565	0.288
C_{13}	21.38	21.767	1.69	1.74	2.840	3.044
C_{14}	21.13	21.345	1.44	1.32	2.069	1.749
C_{15}	18.42	18.424	-1.27	-1.60	1.625	2.555
Total	295.37	300.336			62.168	59.043
Mean	19.69	20.02				

Legend: $\breve{y} = \Sigma y/n$ where y is the response and n, the number of observed data (responses)
y_k is the experimental value (response) ;y_E is the expected or theoretically calculated value(response)

IV. CONCLUSION

The strengths (responses) of the SCBA concrete were a function of the proportions of its ingredients: water, cement, SCBA, fine aggregate and coarse aggregates. Since the predicted strengths by the model were in total agreement with the corresponding experimentally–observed values, the null hypothesis was satisfied. This meant that the model equation was valid.

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