Mahgoub Transform (Laplace-Carson Transform) of Error Function

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Abstract: **When many advanced problems of probability, statistics, physics and engineering like heat conduction problems, vibrating beams problems express mathematically then error function appears frequently in these problems. In this article, we find the Mahgoub transform (Laplace-Carson transform) of error function. In application section, some numerical applications of Mahgoub transform of error function are given for evaluating the improper integral, which contain error function.**

Keywords: **Mahgoub transform, Error function, Complementary error function, Improper integral**

I. INTRODUCTION

Integral transforms play a vital role for solving many advance problems like radioactive decay problems, population growth problems, vibration problems of beam, electric circuit problems and motion of a particle under gravity which appear in many branches of engineering and sciences. Many scholars used different integral transforms (Laplace transform [1], Fourier transform [2], Kamal transform [3-7, 29], Mahgoub transform [8-14, 30-33], Elzaki transform [15-16, 34-35], Aboodh transform [17-20, 36-39], Mohand transform [21-23, 40-43], Sumudu transform [44-45] and Shehu transform [46]) and solved differential equations, partial differential equations, integral equations, integro-differential equations and partial integro-differential equations. Sudhanshu et al. [24-28] discussed the comparative study of these transforms.

Integral transforms are very useful for finding the solutions of engineering problems like heat and mass transfer problems, Fick's second law, vibrating beams problems. The solution of these types of problems contain error and complementary error function when solved by any integral transform so it is very necessary to knowing the integral transforms of error function.

In mathematics, error and complimentary error functions are defined by [47-52]

$$erf(x)=\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

and

$$erfc(x)=\frac{2}{\sqrt{\pi}}\int_x^\infty e^{-t^2}dt \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

In 2016, Mahgoub [30] defined a new integral transform "Mahgoub transform'' of the function $F(t)$ for $t\geq 0$ as

$$M\{F(t)\}=v\int_0^\infty F(t)e^{-vt}dt$$

$$=H(v), k_1\leq v\leq k_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

where operator M is called the Mahgoub transform operator.

The goal of the present article is to determine Mahgoub transform of error function and explain the advantage of Mahgoub transform of error function by giving some numerical applications in application section.

II. SOME USEFUL PROPERTIES OF MAHGOUB TRANSFORM

2.1 Linearity property [8, 12-14]:

If Mahgoub transform of functions $F_1(t)$ and $F_2(t)$ are $H_1(v)$ and $H_2(v)$ respectively then Mahgoub transform of $[aF_1(t)+bF_2(t)]$ is given by

$[aH_1(v)+bH_2(v)]$, where a,b are arbitrary constants.

Proof: By the definition of Mahgoub transform, we have

$$M\{F(t)\}=v\int_0^\infty F(t)e^{-vt}dt$$

$$\Rightarrow M\{aF_1(t)+bF_2(t)\}=v\int_0^\infty [aF_1(t)+bF_2(t)]e^{-vt}dt$$

$$\Rightarrow M\{aF_1(t)+bF_2(t)\} = av\int_0^\infty F_1(t)e^{-vt}dt+bv\int_0^\infty F_2(t)e^{-vt}dt$$

$\Rightarrow M\{aF_1(t)+bF_2(t)\}=aM\{F_1(t)\}+bM\{F_2(t)\}$

$\Rightarrow M\{aF_1(t)+bF_2(t)\}=aH_1(v)+bH_2(v)$,

where a, b are arbitrary constants.

2.2 Change of scale property [14]:

If Mahgoub transform of function $F(t)$ is $H(v)$ then Mahgoub transform of function $F(at)$is given by $H\left(\frac{v}{a}\right)$.

Proof: By the definition of Mahgoub transform, we have

$$M\{F(at)\}=v\int_0^\infty F(at)e^{-vt}dt \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Put $at=p \Rightarrow adt=dp$ in equation (4), we have

$$M\{F(at)\}=\frac{v}{a}\int_0^\infty F(p)e^{\frac{-vp}{a}}dp$$

$$\Rightarrow M\{F(at)\}=H\left(\frac{v}{a}\right).$$

2.3 Shifting property:

If Mahgoub transform of function $F(t)$ is $H(v)$ then Mahgoub transform of function $e^{at}F(t)$ is given by $\frac{v}{(v-a)}H(v-a)$.

Proof: By the definition of Mahgoub transform, we have

$$M\{e^{at}F(t)\}=v\int_0^\infty e^{at}F(t)e^{-vt}dt$$

$$\Rightarrow M\{e^{at}F(t)\}=v\int_0^\infty F(t)e^{-(v-a)t}dt$$

$$=\frac{v}{(v-a)}(v-a)\int_0^\infty F(t)e^{-(v-a)t}dt=\frac{v}{(v-a)}H(v-a).$$

2.4 Mahgoub transform of the derivatives of the function $F(t)$ [9-11, 13-14]:

If $M\{F(t)\}=H(v)$ then

a) $M\{F'(t)\}=vH(v)-vF(0)$

b) $M\{F''(t)\}=v^2H(v)-v^2F(0)-vF'(0)$

c) $M\{F^{(n)}(t)\}=v^nH(v)-v^nF(0)-v^{n-1}F'(0)-\cdots\ldots-vF^{(n-1)}(0)$

2.5 Mahgoub transform of integral of a function $F(t)$:

If $M\{F(t)\}=H(v)$ then

$$M\left\{\int_0^t F(t)dt\right\}=\frac{1}{v}H(v)$$

Proof: Let $G(t)=\int_0^t F(t)dt$. Then

$G'(t)=F(t)$ and $G(0)=0$.

Now by the property of Mahgoub transform of the derivative of function, we have

$$M\{G'(t)\}=vM\{G(t)\}-vG(0)=vM\{G(t)\}$$

$$\Rightarrow M\{G(t)\}=\frac{1}{v}M\{G'(t)\}=\frac{1}{v}M\{F(t)\}$$

$$\Rightarrow M\{G(t)\}=\frac{1}{v}H(v)$$

$$\Rightarrow M\left\{\int_0^t F(t)dt\right\}=\frac{1}{v}H(v)$$

2.6 Mahgoub transform of function $tF(t)$[9]:

If $M\{F(t)\}=H(v)$ then

$$M\{tF(t)\}=\left[\frac{1}{v}-\frac{d}{dv}\right]H(v)$$

Proof: By the definition of Mahgoub transform, we have

$$M\{F(t)\}=v\int_0^\infty F(t)e^{-vt}dt=H(v)$$

$$\Rightarrow \frac{d}{dv}H(v)=\int_0^\infty F(t)e^{-vt}dt+v\int_0^\infty(-t)F(t)e^{-vt}dt$$

$$\Rightarrow \frac{d}{dv}H(v)=\frac{1}{v}.v\int_0^\infty F(t)e^{-vt}dt-v\int_0^\infty tF(t)e^{-vt}dt$$

$$\Rightarrow \frac{d}{dv}H(v)=\frac{1}{v}H(v)-M\{tF(t)\}$$

$$\Rightarrow M\{tF(t)\}=\left[\frac{1}{v}-\frac{d}{dv}\right]H(v)$$

2.7 Convolution theorem for Mahgoub transforms [8, 10-12, 14]:

If Mahgoub transform of functions $F_1(t)$ and $F_2(t)$ are $H_1(v)$ and $H_2(v)$ respectively then Mahgoub transform of their convolution $F_1(t)*F_2(t)$ is given by

$$M\{F_1(t)*F_2(t)\}=\frac{1}{v}M\{F_1(t)\}M\{F_2(t)\}$$

$\Rightarrow M\{F_1(t)*F_2(t)\}=\frac{1}{v}H_1(v)H_2(v)$, where $F_1(t)*F_2(t)$ is defined by

$$F_1(t)*F_2(t)=\int_0^t F_1(t-x)F_2(x)dx$$

$$=\int_0^t F_1(x)F_2(t-x)dx$$

III. MAHGOUB TRANSFORM OF FREQUENTLY ENCOUNTERED FUNCTIONS [8-14]

Table: 1

S.N.	$F(t)$	$M\{F(t)\} = H(v)$
1.	1	1
2.	t	$\frac{1}{v}$
3.	t^2	$\frac{2!}{v^2}$
4.	$t^n, n \in N$	$\frac{n!}{v^n}$
5.	$t^n, n > -1$	$\frac{\Gamma(n+1)}{v^n}$
6.	e^{at}	$\frac{v}{v-a}$
7.	$sinat$	$\frac{av}{v^2+a^2}$
8.	$cosat$	$\frac{v^2}{v^2+a^2}$
9.	$sinhat$	$\frac{av}{v^2-a^2}$
10.	$coshat$	$\frac{v^2}{v^2-a^2}$
11.	$J_0(t)$	$\frac{v}{\sqrt{v^2+1}}$
12.	$J_1(t)$	$v - \frac{v^2}{\sqrt{v^2+1}}$

IV. SOME IMPORTANT PROPERTIES OF ERROR AND COMPLEMENTARY ERROR FUNCTIONS

4.1 The sum of error and complementary error functions is unity:

$erf(x) + erfc(f) = 1$

Proof: we have $\int_0^\infty e^{-t^2} dt = \frac{\sqrt{\pi}}{2}$

$$\Rightarrow \frac{2}{\sqrt{\pi}} \int_0^\infty e^{-t^2} dt = 1$$

$$\Rightarrow \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt + \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2} dt = 1$$

$$\Rightarrow erf(x) + erfc(f) = 1$$

4.2 Error function is an odd function:

$erf(-x) = -erf(x)$

4.3 The value of error function at $x = 0$ is 0:

$erf(0) = 0.$

4.4 The value of complementary error function at $x = 0$ is 1:

$erfc(0) = 1.$

4.5 The domain of error and complementary error functions is $(-\infty, \infty)$.

4.6 $erf(x) \to 1 as\ x \to \infty$.

4.7 $erfc(x) \to 0 as\ x \to \infty$.

4.8 The value of error functions $erf(x)$ for different values of x [48]:

Table: 2

S.N.	x	$erf(x)$
1.	0.00	0.00000
2.	0.02	0.02256
3.	0.04	0.04511
4.	0.06	0.06762
5.	0.08	0.09008
6.	0.10	0.11246
7.	0.12	0.13476
8.	0.14	0.15695
9.	0.16	0.17901
10.	0.18	0.20094
11.	0.20	0.22270

V. MAHGOUB TRANSFORM OF ERROR FUNCTION

By equation (1), we have

$$erf(\sqrt{t}) = \frac{2}{\sqrt{\pi}} \int_0^{\sqrt{t}} e^{-x^2} dx$$

$$= \frac{2}{\sqrt{\pi}} \int_0^{\sqrt{t}} \left[1 - \frac{x^2}{1!} + \frac{x^4}{2!} - \frac{x^6}{3!} + \cdots \ldots\right] dx$$

$$= \frac{2}{\sqrt{\pi}} \left[x - \frac{x^3}{3.1!} + \frac{x^5}{5.2!} - \frac{x^7}{7.3!} + \cdots \ldots\right]_0^{\sqrt{t}}$$

$$= \frac{2}{\sqrt{\pi}} \left[t^{1/2} - \frac{t^{3/2}}{3.1!} + \frac{t^{5/2}}{5.2!} - \frac{t^{7/2}}{7.3!} + ..\right] \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (5)$$

Applying Mahgoub transform both sides on equation$n(5)$, we get

$$M\{erf(\sqrt{t})\} = \frac{2}{\sqrt{\pi}} M\left\{\left[t^{1/2} - \frac{t^{3/2}}{3.1!} + \frac{t^{5/2}}{5.2!} - \frac{t^{7/2}}{7.3!} + ..\right]\right\} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (6)$$

Applying the linearity property of Mahgoub transform on equation (6), we get

$$M\{erf(\sqrt{t})\} = \frac{2}{\sqrt{\pi}} \left[\frac{\Gamma(3/2)}{v^{1/2}} - \frac{\Gamma(5/2)}{v^{3/2}.3.1!} + \frac{\Gamma(7/2)}{v^{5/2}.5.2!} - \frac{\Gamma(9/2)}{v^{7/2}.7.3!} + \cdots\right]$$

$$= \frac{2}{\sqrt{\pi}} \frac{\Gamma(3/2)}{v^{1/2}} \left[1 - \frac{1}{2}\left(\frac{1}{v}\right) + \frac{1.3}{2.4}\left(\frac{1}{v}\right)^2 - \frac{1.3.5}{2.4.6}\left(\frac{1}{v}\right)^3 + \cdots \ldots \ldots\right]$$

$$=\frac{1}{v^{1/2}}\left(1+\frac{1}{v}\right)^{-1/2}=\frac{1}{\sqrt{(1+v)}} \dots\dots\dots\dots\dots\dots\dots\dots\dots (7)$$

VI. MAHGOUB TRANSFORM OF COMPLEMENTARY ERROR FUNCTION

We have, $erf(x)+erfc(f)=1$

$$\Rightarrow erfc(f)=1-erf(x) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (8)$$

Applying Mahgoub transform both sides on equation (8), we have

$$M\{erfc(f)\}=M\{1-erf(x)\} \dots\dots\dots\dots\dots\dots\dots\dots\dots (9)$$

Applying the linearity property of Mahgoub transform on equation (9), we get

$$M\{erfc(f)\}=M\{1\}-M\{erf(x)\}$$

$$\Rightarrow M\{erfc(f)\}=1-\frac{1}{\sqrt{(1+v)}}$$

$$\Rightarrow M\{erfc(f)\}=\left[\frac{\sqrt{(1+v)}-1}{\sqrt{(1+v)}}\right] \dots\dots\dots\dots\dots\dots\dots\dots\dots (10)$$

VII. APPLICATIONS

In this section, we solve some improper integral, which contain error function for explaining the effectiveness of Mahgoub transform of error function.

7.1 Evaluate the improper integral

$$I=\int_0^\infty e^{-t}\, erf\left(\sqrt{t}\right) dt.$$

We have $M\{erf(\sqrt{t})\}=v\int_0^\infty erf(\sqrt{t})\, e^{-vt}\, dt$

$$=\frac{1}{\sqrt{(1+v)}} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (11)$$

Taking $v\to 1$ in above equation, we have

$$I=\int_0^\infty e^{-t}\, erf\left(\sqrt{t}\right) dt=\frac{1}{\sqrt{2}}$$

7.2 Evaluate the improper integral

$$I=\int_0^\infty te^{-3t}\, erf\left(\sqrt{t}\right) dt.$$

We have $M\{erf(\sqrt{t})\}=\frac{1}{\sqrt{(1+v)}}$

$$\Rightarrow M\{t\, erf(\sqrt{t})\}=\left[\frac{1}{v}-\frac{d}{dv}\right]\frac{1}{\sqrt{(1+v)}}$$

$$=\frac{1}{v\sqrt{(1+v)}}+\frac{1}{2(1+v)^{\frac{3}{2}}} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (12)$$

By the definition of Mahgoub transform, we have

$$M\{t\, erf(\sqrt{t})\}=v\int_0^\infty t\, erf\left(\sqrt{t}\right) e^{-vt}\, dt \dots\dots\dots\dots\dots\dots (13)$$

Now by equations (12) and (13), we get

$$v\int_0^\infty t\, erf\left(\sqrt{t}\right) e^{-vt}\, dt=\frac{1}{v\sqrt{(1+v)}}+\frac{1}{2(1+v)^{\frac{3}{2}}}$$

Taking $v\to 3$ in above equation, we have

$$3\int_0^\infty te^{-3t}\, erf\left(\sqrt{t}\right) dt=\frac{1}{6}+\frac{1}{16}=\frac{11}{48}$$

$$I=\int_0^\infty te^{-3t}\, erf\left(\sqrt{t}\right) dt=\frac{11}{144}$$

7.3 Evaluate the improper integral

$$I=\int_0^\infty e^{-(v-2)t}\, erf\left(\sqrt{t}\right) dt.$$

We have $M\{erf(\sqrt{t})\}=\frac{1}{\sqrt{(1+v)}}$

Now by shifting theorem of Mahgoub transform, we have

$$M\{e^{2t}\, erf(\sqrt{t})\}=\frac{v}{(v-2)}\left[\frac{1}{\sqrt{(v-1)}}\right]$$

$$\Rightarrow M\{e^{2t}\, erf(\sqrt{t})\}=\frac{v}{(v-2)\sqrt{(v-1)}} \dots\dots\dots\dots\dots\dots (14)$$

By the definition of Mahgoub transform, we have

$$M\{e^{2t}\, erf(\sqrt{t})\}=v\int_0^\infty e^{2t}\, erf\left(\sqrt{t}\right) e^{-vt}\, dt$$

$$\Rightarrow M\{e^{2t}\, erf(\sqrt{t})\}$$
$$=v\int_0^\infty e^{-(v-2)t}\, erf\left(\sqrt{t}\right) dt \dots\dots\dots (15)$$

Now by equations (14) and (15), we get

$$v\int_0^\infty e^{-(v-2)t}\, erf\left(\sqrt{t}\right) dt=\frac{v}{(v-2)\sqrt{(v-1)}}$$

$$\Rightarrow I=\int_0^\infty e^{-(v-2)t}\, erf\left(\sqrt{t}\right) dt=\frac{1}{(v-2)\sqrt{(v-1)}}.$$

7.4 Evaluate the improper integral

$$I=\int_0^\infty e^{-t}\left\{\int_0^t erf\left(\sqrt{u}\right) du\right\} dt.$$

We have $M\{erf(\sqrt{t})\}=\frac{1}{\sqrt{(1+v)}}$

Now by the property of Mahgoub transform of integral of a function, we have

$$M\left\{\int_0^t erf(\sqrt{u})du\right\} = \frac{1}{v}\left[\frac{1}{\sqrt{(1+v)}}\right]$$

$$\Rightarrow M\left\{\int_0^t erf(\sqrt{u})du\right\} = \frac{1}{v\sqrt{(1+v)}} \dots\dots\dots\dots\dots (16)$$

By the definition of Mahgoub transform, we have

$$M\left\{\int_0^t erf(\sqrt{u})du\right\} = v\int_0^\infty e^{-vt}\left\{\int_0^t erf(\sqrt{u})du\right\}dt \dots (17)$$

Now by equations (16) and (17), we get

$$v\int_0^\infty e^{-vt}\left\{\int_0^t erf(\sqrt{u})du\right\}dt = \frac{1}{v\sqrt{(1+v)}}$$

Taking $v \to 1$ in above equation, we have

$$I = \int_0^\infty e^{-t}\left\{\int_0^t erf(\sqrt{u})du\right\}dt = \frac{1}{\sqrt{2}}.$$

7.5 Evaluate the improper integral

$$I = \int_0^\infty e^{-2t}\left[\frac{d}{dt}erf(2\sqrt{t})\right]dt.$$

We have $M\{erf(\sqrt{t})\} = \frac{1}{\sqrt{(1+v)}}$

Now by change of scale property of Mahgoub transform, we have

$$M\{erf(2\sqrt{t})\} = \left[\frac{1}{\sqrt{(1+v/4)}}\right]$$

$$\Rightarrow M\{erf(2\sqrt{t})\} = \frac{2}{\sqrt{(4+v)}}$$

Now using the property of Mahgoub transform of derivative of a function, we have

$$M\left\{\frac{d}{dt}erf(2\sqrt{t})\right\} = v.\frac{2}{\sqrt{(4+v)}} - v.0$$

$$\Rightarrow M\left\{\frac{d}{dt}erf(2\sqrt{t})\right\} = \frac{2v}{\sqrt{(4+v)}} \dots\dots\dots\dots\dots\dots (18)$$

By the definition of Mahgoub transform, we have

$$M\left\{\frac{d}{dt}erf(2\sqrt{t})\right\}$$
$$= v\int_0^\infty e^{-vt}\left\{\frac{d}{dt}erf(2\sqrt{t})\right\}dt \dots\dots\dots\dots\dots\dots\dots (19)$$

Now by equations (18) and (19), we get

$$v\int_0^\infty e^{-vt}\left\{\frac{d}{dt}erf(2\sqrt{t})\right\}dt = \frac{2v}{\sqrt{(4+v)}}$$

Taking $v \to 2$ in above equation, we have

$$2\int_0^\infty e^{-2t}\left\{\frac{d}{dt}erf(2\sqrt{t})\right\}dt = \frac{4}{\sqrt{(6)}}$$

$$\Rightarrow I = \int_0^\infty e^{-2t}\left\{\frac{d}{dt}erf(2\sqrt{t})\right\}dt = \frac{2}{\sqrt{(6)}}$$

$$\Rightarrow I = \int_0^\infty e^{-2t}\left\{\frac{d}{dt}erf(2\sqrt{t})\right\}dt = \sqrt{\frac{2}{3}}.$$

7.6 Evaluate the improper integral

$$I = \int_0^\infty e^{-5t}\left[erf\sqrt{t} * erf\sqrt{t}\right]dt.$$

By convolution theorem of Mahgoub transform, we have

$$M\{erf\sqrt{t} * erf\sqrt{t}\} = \frac{1}{v}M\{erf(\sqrt{t})\}M\{erf(\sqrt{t})\}$$

$$= \frac{1}{v}\left[\frac{1}{\sqrt{(1+v)}}\right]\left[\frac{1}{\sqrt{(1+v)}}\right] = \frac{1}{v(1+v)}.\dots\dots\dots\dots\dots (20)$$

Now by the definition of Mahgoub transform, we have

$$M\{erf\sqrt{t} * erf\sqrt{t}\}$$
$$= v\int_0^\infty e^{-vt}\{erf\sqrt{t} * erf\sqrt{t}\}dt \dots\dots\dots\dots\dots\dots\dots (21)$$

Now by equations (20) and (21), we get

$$v\int_0^\infty e^{-vt}\{erf\sqrt{t} * erf\sqrt{t}\}dt = \frac{1}{v(1+v)} \dots\dots\dots\dots (22)$$

Taking $v \to 5$ in above equation, we have

$$5\int_0^\infty e^{-5t}\{erf\sqrt{t} * erf\sqrt{t}\}dt = \frac{1}{30}$$

$$\Rightarrow I = \int_0^\infty e^{-5t}\{erf\sqrt{t} * erf\sqrt{t}\}dt = \frac{1}{150}.$$

VIII. CONCLUSIONS

In this article, we have successfully discussed the Mahgoub transform of error function. The given numerical applications in application section show the advantage of Mahgoub transform of error function.

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