

Graphic Design Generation of Gear Profile

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Abstract— Gears are a key component that is widely used in various rotating equipment in order to transmit power and change speed. Spur involute gears are in line contact at every instant, and therefore they are sensitive to the misalignment of gear axes. For this reason, it is necessary to localize their bearing contact, and this can be achieved by crowning the surface of one of the mating gears. It is preferable to crown the pinion tooth surface rather than the gear tooth surface because the number of pinion teeth is smaller than the number of gear teeth. The tooth profile of the spur gears is generated as an involute curve. The structure of this research is to design gear model in MATLAB, the first step is to find out the functions and the relationships of all the curves of the gear from mathematic and geometric point of view, and then use proper MATLAB codes and functions to programme it. So, after establishing the perfect involute gear models by MATLAB codes, MATLAB can offer a simple and convenient way to implement the tooth profile modification codes with the established gear models codes to obtain all kinds of modified gear models. Besides, for the tooth profiles are consisted by coordinates, users can decrease the gap between each nodes to improve the accuracy of modification.

Keywords— Gear, Involute Tooth, MATLAB Codes

I. GEOMETRY OF INVOLUTE CURVES

Consider the particular case when the evolute E is a circle. The involute I for such a case is the tooth profile for a spur gear. The evolute, the circle of radius r_b in Figure 1, is called the *base circle*. Two branches of an involute curve are shown in Figure 1. They are generated by point M_o of the straight line that rolls over the base circle clockwise and counterclockwise, respectively. Each branch represents its respective side of the tooth in Figure 2.

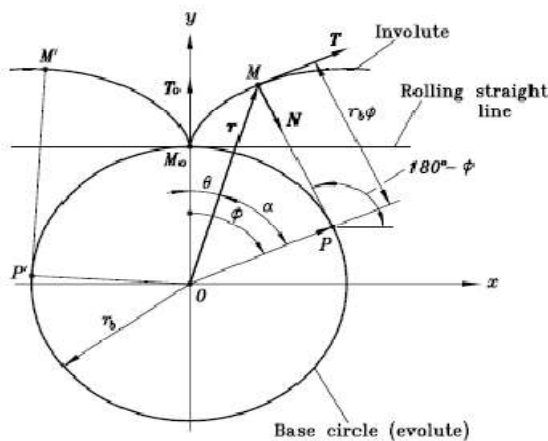


Fig1. Involute curve

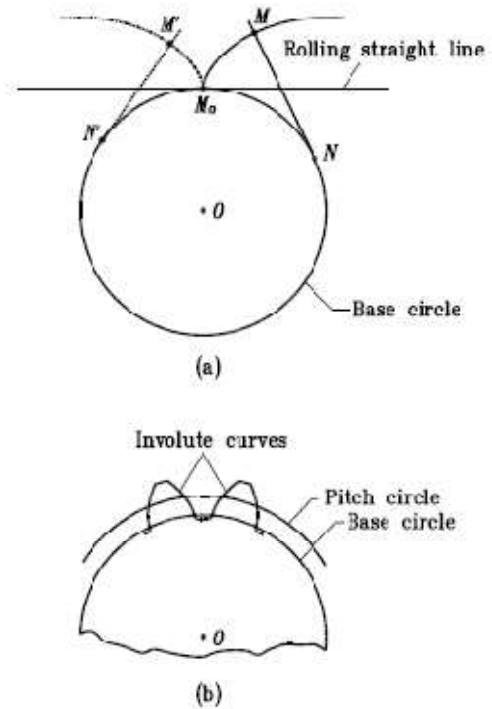


Fig2. Two branches of an involute curve

The analytical representation of an involute curve is based on the following considerations in Figure 1. A current point M of the involute curve is determined by the vector equation.

$$\overline{OM} = \overline{OP} + \overline{PM} \quad (1)$$

$$\overline{OP} = r_b [\sin\phi \quad \cos\phi]^T \quad (2)$$

$$\overline{PM} = PM [-\cos\phi \quad \sin\phi]^T \quad (3)$$

Due to rolling without sliding,

$$PM = M_o \hat{P} = r_b \phi \quad (4)$$

Here, ϕ is the angle of rotation in rolling motion.

Equation 1 to 4 yield,

$$x = r_b (\sin\phi - \phi \cos\phi), \quad y = r_b (\cos\phi + \phi \sin\phi) \quad (5)$$

Another representation of an involute curve is based on application of variable parameter α (Fig. 1). The derivation of equations of the involute curve may be accomplished as follows:

$$x = r \sin \theta, y = r \cos \theta \tag{6}$$

$$MP = r_b \tan \alpha \quad \theta = \tan \alpha - \alpha \tag{7}$$

Function $\theta(\alpha)$ is designated as $\text{inv } \alpha$. Equations (1) and (7) yield

$$x = \frac{r_b}{\cos \alpha} \sin(\text{inv } \alpha), \quad y = \frac{r_b}{\cos \alpha} \cos(\text{inv } \alpha) \tag{8}$$

Function

$$\text{inv } \alpha = \tan \alpha - \alpha \tag{9}$$

may be determined by direct computation considering α as given. The *inverse* operation, determination of α considering $\text{inv } \alpha$ as given, needs the solution of the nonlinear equation,

$$\alpha - \tan \alpha + \text{inv } \alpha = 0 \tag{10}$$

II. EXTENDED AND SHORTENED INVOLUTE CURVES

These curves are traced out by point M , which is offset with respect to the rolling straight line (Figs. 3 and 4). The straight line rolls over the circle of radius r_b . Using an approach similar to that discussed above, we obtain the following equations:

$$\begin{aligned} x &= (r_b \mp h) \sin \phi - r_b \phi \cos \phi \\ y &= (r_b \mp h) \cos \phi - r_b \phi \sin \phi \end{aligned} \tag{11}$$

The upper sign in Eqs. (11) corresponds to the *extended* involute (Fig. 3), and the lower sign to the *shortened* involute (Fig. 4).

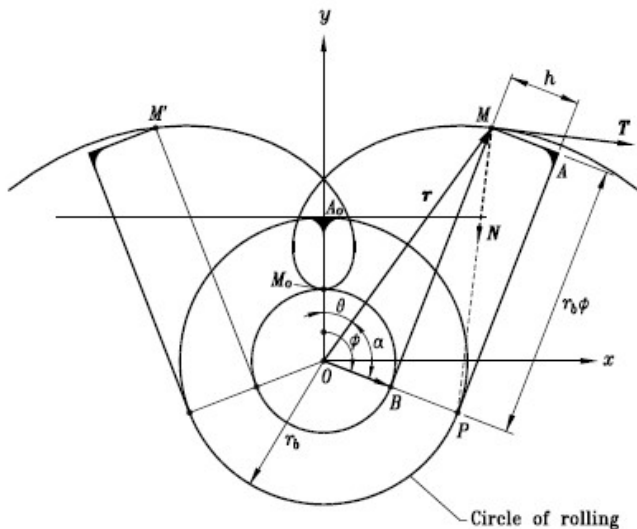


Fig 3. Extended Involute Curve

Parameter h is the offset of the tracing point M with respect to the rolling straight line. Two branches are generated by rolling of the straight line clockwise and counterclockwise, respectively.

The common point of the two branches is M_0 , and M_0 is a regular curve point. Point P is the instantaneous center of rotation of the rolling straight line.

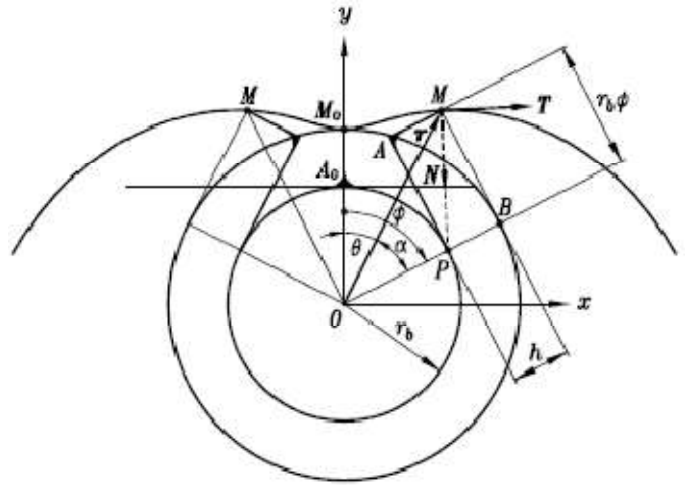


Fig 4. Shortened Involute Curve

III. GENERATION OF INVOLUTE CURVES

The generation of a spur involute gear by a rack-cutter is shown in Fig. 5. The velocity v and angular velocity ω are related by the equation,

$$\frac{v}{\omega} = r_p = \frac{N}{2P} \tag{12}$$

Here, r_p is the radius of the pitch circle, N is the number of gear teeth, and P is the diametral pitch. The base circle of radius r_b (Fig. 5) determined as

$$r_b = \frac{N \cos \alpha_c}{2P} \tag{13}$$

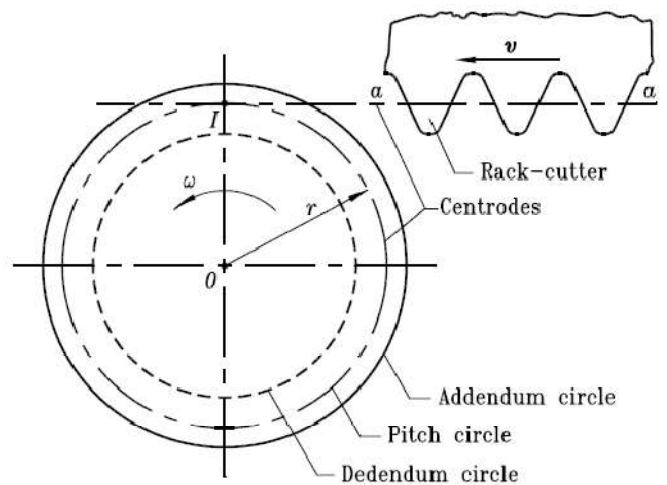


Fig5. Generation of Involute Curve

IV. TOOTH ELEMENT PROPORTIONS

The pitch circle is the reference circle for tooth element proportions (Fig. 6).

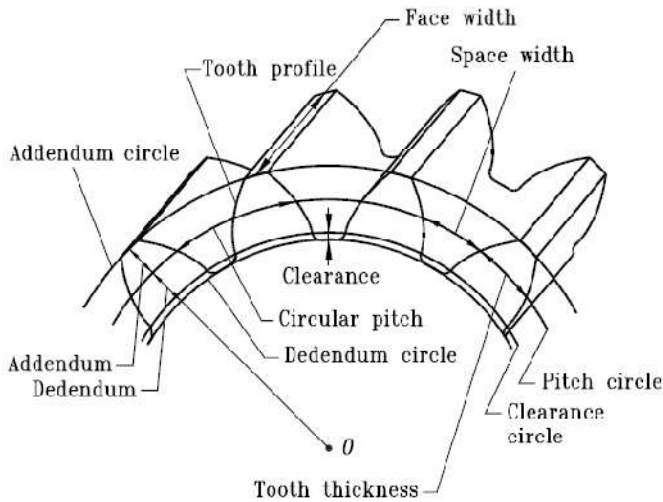


Fig6. Gear Tooth Parameters

The circumference of the pitch circle is

$$L = p_c N = \pi d \quad (14)$$

where N is the number of gear teeth.

The *module* m is represented as

$$m = \frac{p_c}{\pi} = \frac{d}{N} \quad (15)$$

The *addendum* a is the *radial* distance between the addendum circle and the pitch circle.

The *dedendum* b is the *radial* distance between the pitch circle and the dedendum circle.

The tooth thickness t and the space width w are arcs measured *along* the pitch circle.

V. INVOLUTE CURVE AND GEAR TOOTH PROFILE

An involute curve may be generated as shown in Fig. 7. A partial flange B is attached to the cylinder A , around which is wrapped a cord def , which is held tight. Point b on the cord represents the tracing point, and as the cord is wrapped and unwrapped about the cylinder, point b will trace out the involute curve ac . The radius of the curvature of the involute varies continuously, being zero at point a and a maximum at point c . At point b the radius is equal to the distance be , since point b is instantaneously rotating about point e . Thus the generating line de is normal to the involute at all points of intersection and, at the same time, is always tangent to the cylinder A . The circle on which the involute is generated is called the *base circle*.

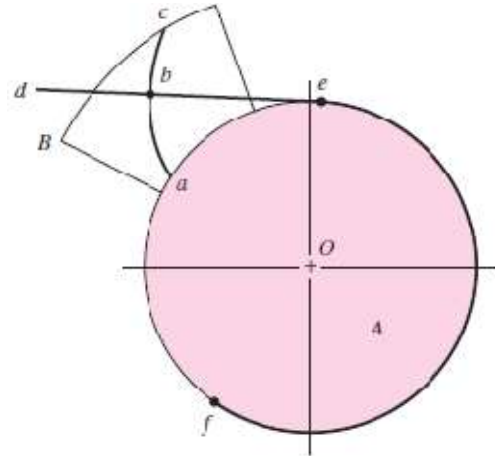


Fig7. Generation of An Involute

Among other things, it is necessary that we actually be able to draw the teeth on a pair of meshing gears. Rather, we make drawings of gear teeth to obtain an understanding of the problems involved in the meshing of the mating teeth.

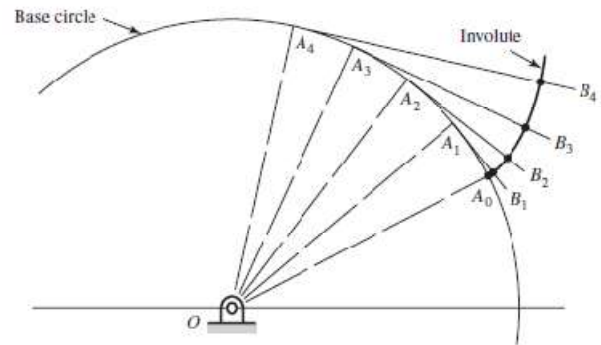


Fig8. Construction of An Involute Curve

VI. RESULTS AND DISCUSSION

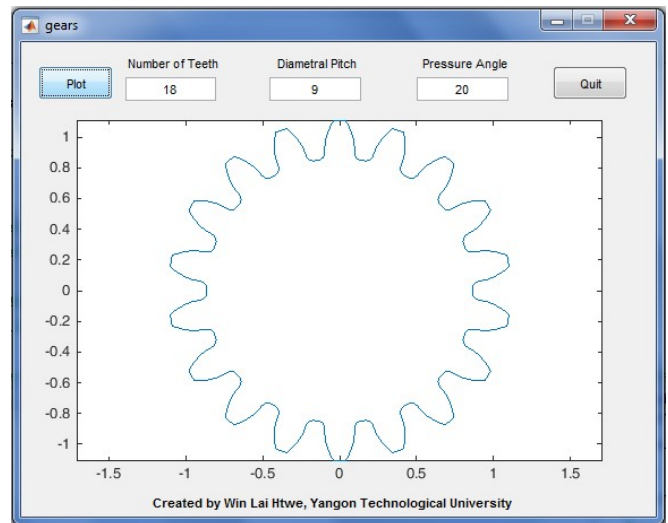


Fig9. Two Dimensional Whole Gear Involute Profile of Tooth

Computation of the pitch circle radius, addendum circle radius, base circle radius for the base circle radius, circular pitch of the tooth and tooth thickness along pitch circle were done to obtain the complete two-dimensional whole gear involute tooth profile from MATLAB.

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