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Mathematical Model for Two Wheeled Balancing Robot Using PID Control Algorithm

Thet Naing Soe[#], Aye Su Khaing[*]

[#]*Department of Mechatronic Engineering, Technological University (Mawlamyine)*
[*] *Department of Electronic Engineering, Technological University (Mawlamyine)*

***Abstract*—Robots are able to move themselves anywhere and they can carry out a variety of applications. Designing a mobile robot with special capabilities has become a trend these days and also fits well with the needs and nature of the human lifestyle. This research is focused on the balancing and stability control of two wheeled robot basically lie on the inclination angle. A two wheeled balancing robot can be controlled by using tilt angle data from accelerometer and gyroscope. The purpose of complementary filter is implemented in fusion of gyroscope and accelerometer readings to obtain more accurate tilt angle measurement of the robot. The PID controller is implemented in the robot control system to control the speed of the motor based on the feedback tilt angle data from complementary filter for balancing purpose.**

***Keywords*—balancing, stability, tilt angle, complementary filter, PID controller.**

I. INTRODUCTION

A two-wheeled balancing robot comprises of three main parts including sensor, microcontroller, driver module and motors. Components of the robot include a core processing unit, inertial measurement unit (IMU) sensor, wheels, power source and associated printed circuit board circuitry modules. A microcontroller in which the arrangement of PID controller and complementary filter provides a type of feedback signal through PWM controls to turn the DC gear motor clockwise or anticlockwise. A microcontroller provides the computational power to allow the robot to balance itself, based on the sensor input information. Complementary filter is one type that provides a best approximation from the multiple sensor inputs, allowing the devices inherent inaccuracies to be overcome. An IMU sensor, which combines accelerometer and gyroscope, is needed to provide the angle position of the inverted pendulum or robot base and input into the controller, which the program itself is a balancing algorithm.

II. METHODOLOGY

This research is focused on the balancing control of two wheeled robot. The basics of the research basically lie on the inclination angle. A two wheeled balancing robot is made by using tilt angle data from accelerometer and gyroscope. The complementary filter is implemented in fusion of gyroscope and accelerometer readings to obtain more accurate tilt angle measurement of the robot. The PID controller is implemented in the robot control system to control the speed of the motor based on the tilt angle data from complementary filter for balancing purpose. The main microcontroller provides the computational power to allow the robot to balance itself, based on the sensor input information. The tangible sections were the robot's hardware and the intangible sections were the robot's control systems or software. These two were integrated together to form a drivable unit. The overall block diagram of balancing control algorithm is shown in Fig.1.

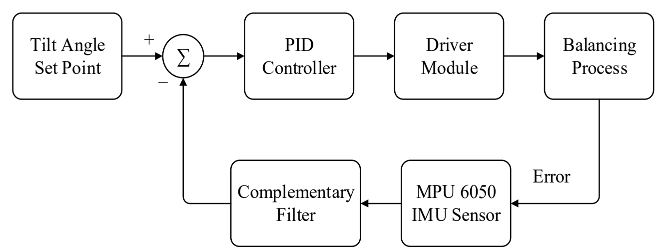

Fig.1Block diagram of balancing control algorithm

III. HARDWARE STRUCTURE AND CIRCUIT DESIGN

The robot chassis is one of the key features of this balancing system because it is the part that will host most components of the robot. It is designed in way that would enable sensor, battery pack, controller unit, driver module and other wiring connections are mounted on its frame without congestion. The chassis is made in such a way that it resembled a layered cabinet where all the components would fit neatly within.

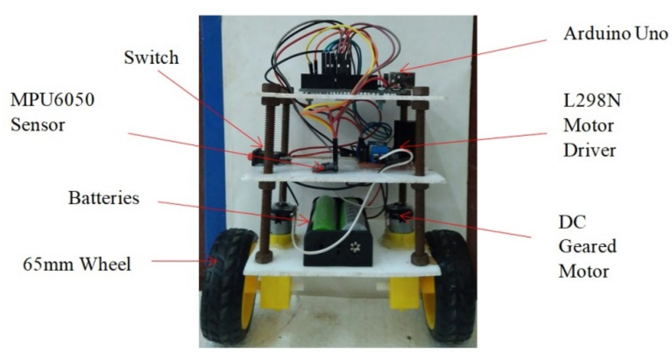

Fig.2 Hardware configuration of two wheeled balancing robot

Plastic is chosen to build the chassis because it is lightweight and also it is a non-electrical conductor reducing the chances of electrical components short circuiting through its base. The bottom plastic plate of the chassis is connected with two semicircular brackets and machine screws for each motor and then the rest of the plastic is connected by four threaded steel wires with nuts. The chassis is rectangular in shape and its dimensions measured approximately 11.5cm length, 9cm width and 15cm high, and weight is about 465grams including the wheels. The picture of the chassis is shown in Fig.2.

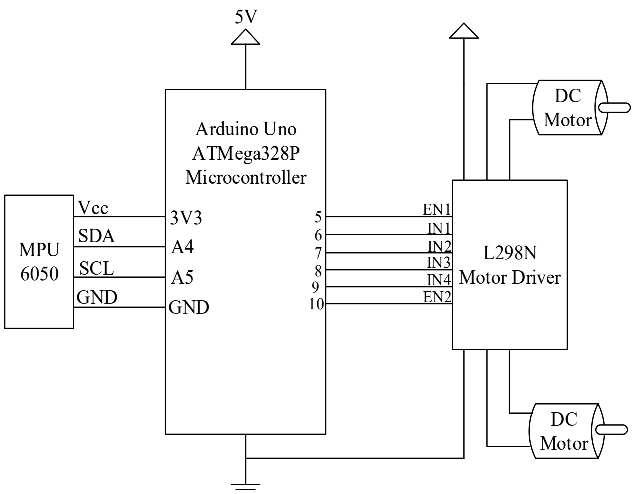

Fig.3 Proposed control circuit design for two wheeled balancing robot

The proposed control circuit designed is shown in Fig. 3. The L298N Motor driver module is directly powered through the 12V terminal pin. The Arduino is powered from 5V output pin of motor driver module. The on-board regulator on the Arduino board, regulate the input 5V and ATmega IC and MPU5060 will be powered by it. The DC motor can run from voltage 5V to 12V. But will be connecting the 8.4V positive wire from battery to 12V input terminal of motor driver module. This will make the motor operate with 8.4V. The following will list how the MPU6050 and L289N motor driver module is connected with Arduino. The MPU6050 communicates with Arduino through I2C interface, so use the SPI pins A4 and A5 of Arduino. The DC motors are connected to PWM pins D7, D6, D8 and D9 respectively. It is need to connect them to PWM pins because will be controlling the speed of the DC motor by varying the duty cycle of the PWM signal.

IV. SOFTWARE PORTION OF BALANCING CONTROL SYSTEM

The balancing robot control system is predefined according to the system flow chart as shown in Fig.4. The Arduino Uno microcontroller is applied to interpret a collection of sensors, and carried out a series of logical sequences or processes that dictated the speed and direction of the motor. In this control system, all the operations are done by the Arduino IDE platform using C++ programming language.

The I/O ports are initialized as follows. The PWM pins were activated as output. The line 'Wire.begin()' started for the I2C. The command line 'Serial.begin(115200)' started sending and receiving serial data. The line 'accelgyro.Initialize()' started the code that would read the IMU sensor. The setup also started the regulators with line 'filter()' and 'myPID'. It also initiated 'timed actions' which were functions that automatically run methods in a regular interval, regardless of what else was happening in the code. The methods used in timed actions in this sketch for updating the sensors.

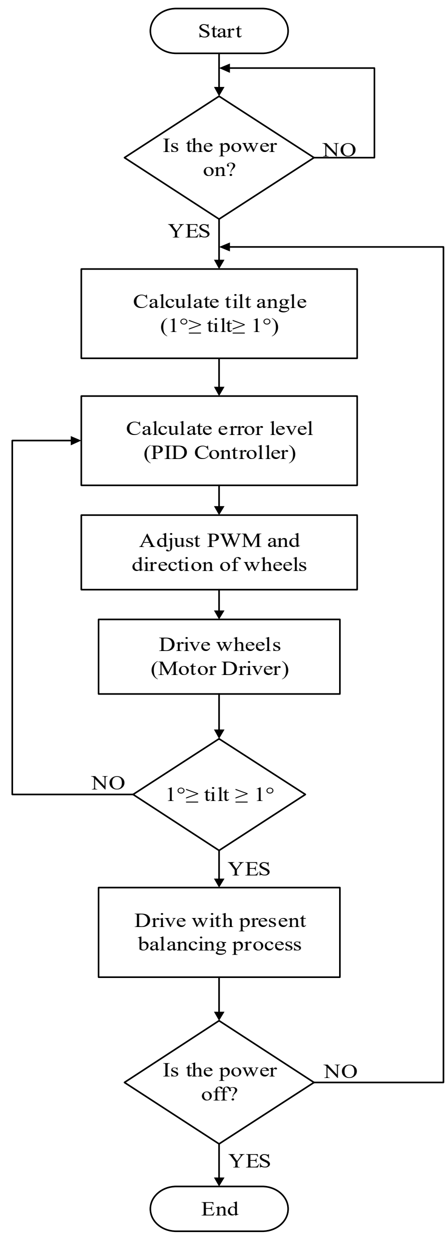

Fig.4 System flow chart for two wheeled balancing robot

In the loop, if the robot was started, it will compute the tilt angle of the robot. If the tilt angle of the robot was out of the range of ±45º, then the robot was falling down. If the tilt angle of the robot was in the range of ±45º, then the filter started working to get the more actuate tilt angle from the sensor fusion. And also the PID controller measured a process variable that was the tilt angle of the robot and calculated an error value that was angle from the vertical which was the deviation of the process variable from some desired, ideal value that was ± 1 º degree from the vertical. The controller attempted to minimize this error over time by continuously adjusting a control variable that is motor torque.

V. MATHEMATICAL MODEL FOR TWO WHEELED ROBOT

The dynamics of the robot is to be described by a mathematical model in order to facilitate the development of an efficient control system for the two wheeled balancing robot. The robot's mathematical description is divided in three parts, one for the electrical motor system, one for the inverted pendulum and one for the wheels. The pendulum and the wheel have three equations each, one for rotational direction and two for x and y-direction. The axes are fixed in the wheels and the wheels are followed when the robot is moving.

A. Mathematical Model of the DC Motor

There may be considered as two main descriptions for the DC Motor. They are mechanical description and electrical description of mathematical modeling.

$$V_s = V_a + V_{emf}$$

The relationship of the shaft rotation, armature current, back emf voltage and motor torque are substituted in this equation.

$$V_s = R_a i_a + K_e \dot{\theta}_m$$

By substituting parameters of DC motor,

$$K_t \frac{V_s - K_e \dot{\theta}_m}{R_a} - b\dot{\theta}_m - \tau_w = J_m \ddot{\theta}_m$$

Laplace transformed to this equation for the transfer function of DC motor

$$K_t \frac{V_s(s) - K_e \theta_m s(s)}{R_a} - b\theta_m s(s) - \tau_w(s) = J_m \theta_m s^2(s) \qquad (1)$$

Where,

K_t = Motor torque constant [Nm/A]
i_a = Armature current [A]
V_{emf} = Back emf voltage [V]
$\dot{\theta}_m$= Angular velocity of the motor shaft [rad/s]
R_a= Armature resistance
b = friction [Nm]
J_m = Inertia of the motor shaft [mr^2]
τ_w= Torque on the rotor inflicted by the wheel [Nm]
$\ddot{\theta}_m$= Angular acceleration of the motor shaft [rad/s^2]
θ_m= Angular position of the rotor
V_s = Supply voltage
K_e = Back emf constant [Vs/rad]

B. Mathematical Model of the Wheel

The motor applies a torque when a current applies and the pendulum generates force when it leaves its marginal stability and the gravity starts pulling it downward. The wheel is effected by forces in one for rotational direction and two for x and y-direction. Using Newton's law of motion, the sum of force around the center of the wheel is in this equation.

$$\tau_w = J_w \ddot{\theta}_w - f_r r$$

Substituting parameters of the wheel,

$$\tau_w = J_w \ddot{\theta}_w - F_{pH} r_w + m_w \ddot{\theta}_w r_w^2$$

Laplace transformed to this equation for the transfer function of the wheel,

$$\tau_w(s) = J_w \theta_w s^2(s) - F_{pH}\, r_w(s) + m_w \theta_w\, r_w^2 s^2(s)(2)$$

Where,

f_r = Friction force
J_w = Inertia of the wheel [$m_w r^2$]
m_w = Mass of the wheel [kg]
F_{pH} = Horizontal force created by the pendulum [N]

C. Mathematical Model of the Inverted Pendulum

There are some different extra efforts while building the model of the robot. The forces which effected to the pendulum are the gravity pulling the mass of the pendulum down, and the motor torque which makes it swing and move back and front. The motor will not have full control over the body, as the wheels are able to move freely.

$$-F_{pH} = m_p \vec{a}_{x,g}$$

Where,

$$\vec{a}_{x,g} = \ddot{x} + l_g \ddot{\theta}_p$$

The angular rotational force is transformed into linear motion by substituting the equation,

$$-F_{pH} = m_p \ddot{\theta}_w r + m_p l_g \ddot{\theta}_p$$

Laplace transformed to this equation for the transfer function of the horizontal force created by the pendulum,

$$F_{pH}(s) = m_p \theta_w r s^2(s) + m_p l_g \theta_p s^2(s)(3)$$

By Newton's law of motion, sum of the movement around the center of mass of the pendulum is

$$F_{pH} l_g = J_p \ddot{\theta}_p$$

By substituting the parameters of horizontal force centered by pendulum in this equation,

$$\ddot{\theta}_w = \frac{J_p\ddot{\theta}_p + m_p l_g^2 \ddot{\theta}_p}{-m_p \tau_w l_g}$$

Laplace transformed to this equation for the transfer function,

$$\theta_w s^2(s) = \left[\frac{J_p\theta_p s^2 + m_p l_g^2\,\theta_p s^2}{-m_p\tau_w l_g}\right](s)(4)$$

Where,

m_p = Mass of the pendulum [kg]

$\vec{a}_{x,g}$= Acceleration of the gravity [m/s]

l_g = Distance from pivot to the gravitational center of the pendulum

θ_p = Angle of the pendulum

$\ddot{\theta}_p$= Pendulum acceleration

$\ddot{\theta}_w$= Wheel acceleration

D. Relationship between Motor Gear Ratio and Wheel

The motor shaft is connected to the wheel, but in between there is a gearing which reduces the load on the motor from the wheels. This is the reason to why θ_m and θ_w are not equal. So the relationship between θ_m and θ_w are described by thus Laplaced equation,

$$\theta_m(s) = \frac{\theta_w(s)}{G_r}(5)$$

Where,

G_r = Gear ratio

E. Final Transfer Function for Two Wheeled Balancing Model

By combining the equations from the divided mathematical models, the complete transfer function of the whole system will emerged. The start was with the mathematical model of the wheel calculated in equation 2, which holds an unknown FpH. The value of horizontal force created by the pendulum FpH is found in equation 3. The mathematical model of the motor is found in equation 1. The final transfer function is calculated as follow.

$$\frac{\theta_p}{V_s}(s) = \frac{\frac{K_t}{R_a}\,(m_p\, l_g\, r_w)}{\left[-\left(J_p - m_p\, l_g^2\right)\left[\frac{J_m}{G_r} + J_w + m_p r_w^2 + m_w r_w^2 + m_p l_g r_w\right]\right]s^2 - \left[\left(J_p - m_p\, l_g^2\right)\left[\frac{K_t\, K_e}{R_a\, G_r} + \frac{b}{G_r}\right]\right]s}$$

By substituting the measured parameters of two wheeled balancing robot in this equation, the final complete transfer function is as follow;

$$\frac{\theta_p}{V_s}(s) = \frac{1}{0.00428s^2 - 0.00116s}$$

VI. Gains and Responses for Two Wheeled Robot

The dynamics of the robot is to be described by a mathematical model in order to facilitate the development of an efficient control system for the two wheeled balancing robot. The robot's mathematical description is divided into three parts, electrical motor system, inverted pendulum and the wheels. By combining the equations from the divided mathematical models, the complete transfer function of the whole system IS emerged.

PID control of two wheeled balancing robot was shown in Fig.5. In order to design a particular PID control loop, the constants in the PID controller have to be adjusted to arrive at acceptable performance. Gains values of PID controller can be tuned manually or automatically in Matlab software to obtain the desired response.

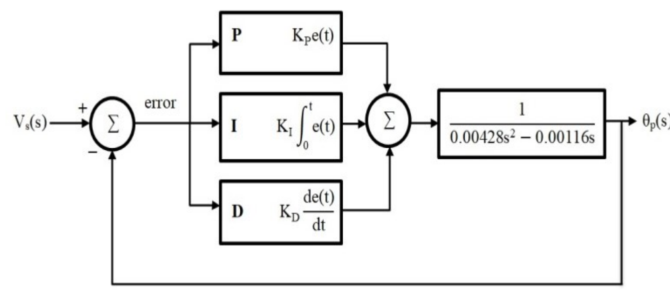

Fig.5 PID control for two wheeled balancing robot

By comparing the responses calculated by control parameters of mathematical models, the gains KP = 18, KI = 0.12 and KD = 538 were the best PID data to satisfy for the design requirements of the control system as shown in Fig.6. It was faster in rise time and reduced the overshoot and improved the system stability.

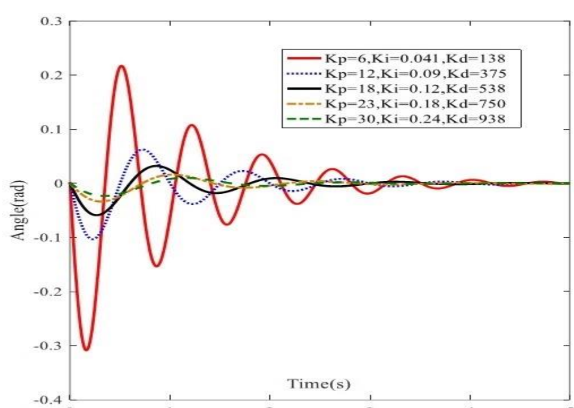

Fig.6 PID Responses for Two Wheeled Balancing Robot

VII. Measurement results for two wheeled robot

The overall testing conditions and monitoring results are completed and finished well. The implementation of the complementary filter in fusing the readings of both accelerometer and gyroscope of the IMU sensor has resulted in more accurate tilt angle measurement. Then, the implements of the PID controller in the robot control system allows more quickly and effective compensation of error between the desired set point and the actual tilt angle thus stabilizes the position of the robot, making it balances nicely on difference surfaces. Measurement monitoring and testing results are shown in following figures.

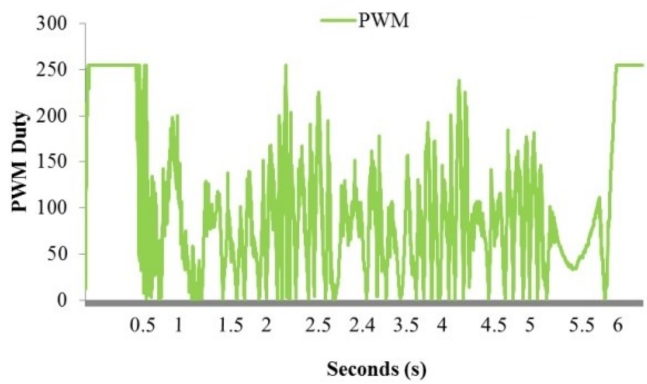

Fig.7PWM output for two wheeled balancing robot

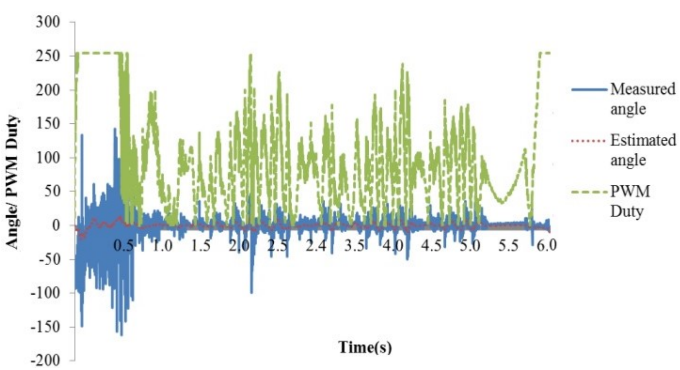

Fig.8PWM output for two wheeled balancing robot

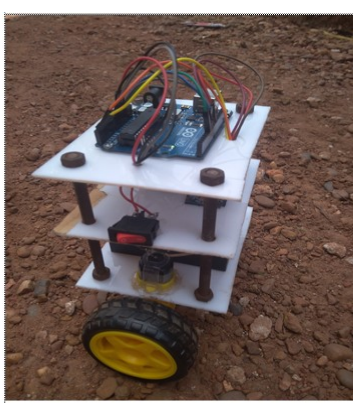

Fig.8 Balance test result for two wheeled balancing robot

VIII. Conclusion

The development of the circuits which involves the specified voltage to be used, signal conditioning, and accurate connectivity had successfully done. The implementation of the PID controller has successfully improved the response in terms of speed and effectiveness of the robot in compensating the error between the set point and the actual tilt angle by controlling the DC motors accordingly.Signal readings from both the accelerometer and gyroscope are more stable and accurate tilt angle measurement. As the result, the robot is able to balance at upright position with the desire data easily on difference surfaces.

Acknowledgment

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References

[1]. Toni Silfver: Control Synthesis for Balancing Robots, Master Thesis of School of Electrical Engineering, (2018). ctions on Automatic Control 30, no.6: 531-541.

[2]. Abdul Gafar: Self-Balancing Robot, Third Year Individual Project of School of Electrical and Electronic Engineering, Manchester (2017).

[3]. Awadalla Taifour Ali: Design And Implementation Of Two Wheeled Self Balancing Robot Using PID Controller, Sudan University of Science and Technology College of Engineering, Sudan, (2017).

[4]. Soe Soe Htay: Balancing Control of Two Wheeled Robot Using PID Controller, M.E Thesis of MTU, Myanmar, (2017).

[5]. Unluturk. U. G. A., Aydogdu. O., 2013. Design and PID Control of Two Wheeled Autonomous Balanced Robot (2013). Report No.38, Department of Electronics Engineering, Techinical University of Selcuk, Turkey.

[6]. Dan Nguyen, Kayan Phuong: A Self-Balancing Robot, KTH Industrial Engineering and Management, (2016).

[7]. Advanced System Dynamics and Control: Modeling, Simulation and Fabrication of a Balancing Robot, 2.151, USA, (2012).