Sawi Transform for Population Growth and Decay Problems

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***Abstract*: In recent years, many scholars have paid attention to find the solution of advance problems of biology, physiology, medicine, economics, engineering and physical sciences by using integral transforms. In this paper, Sawi transform is used for population growth and decay problems. These problems have much importance in the field of economics, chemistry, biology, physics, social science and zoology. We have given some numerical applications to explain the effectiveness of Sawi transform for population growth and decay problems. Results prove that Sawi transform is quite useful for finding the solution of population growth and decay problems.**

***Keywords*: Sawi transform, Inverse Sawi transform, Population growth problem, Decay problem, Half-life.**

***AMS Subject Classification*: 35A22, 65R10, 44A05, 49K15.**

I. INTRODUCTION

Integral transforms methods (Laplace transform [1-2], Fourier transform [2], Mahgoub transform [3-11, 46-48], Kamal transform [12-19, 49], Aboodh transform [20-25, 50-54], Mohand transform [26-36, 45, 55-57], Elzaki transform [37-40, 58-60], Shehu transform [41-43, 61] and Sumudu transform [44, 62-63]) are convenient mathematical tools for solving advance problems of sciences and engineering which are expressible in terms of differential equations, delay differential equations, system of differential equations, partial differential equations, integral equations, system of integral equations, partial integro-differential equations and integro differential equations.

Sawi transform of the function $F(t), t \geq 0$ was proposed by Mahgoub, in 2019 [64] as:

$$S\{F(t)\} = \frac{1}{v^2}\int_0^\infty F(t)e^{-\frac{t}{v}}dt$$

$= R(v), 0 < k_1 \leq v \leq k_2$, where operator S is called the Sawi transform operator.

The Sawi transform of the function $F(t)$ for $t \geq 0$ exist if $F(t)$ is piecewise continuous and of exponential order. The mention two conditions are the only sufficient conditions for the existence of Sawi transforms of the function $F(t)$. He developed this transform for solving linear ordinary differential equations with constant coefficients.

The growth of the population (a plant, or a cell, or an organ, or a species) is mathematically expressed in terms of a first order ordinary linear differential equation [65-69] as

$$\frac{dQ}{dt} = KQ \qquad (1)$$

with initial condition $Q(t_0) = Q_0$ (2)

where K is a positive real number, Q is the amount of population at time t and Q_0 is the initial population at time $t = t_0$.

Equation (1) is known as the Malthusian law of population growth.

The decay problem of the substance is defined mathematically by the following first order ordinary linear differential equation [66, 69] as

$$\frac{dQ}{dt} = -KQ \qquad (3)$$

with initial condition $Q(t_0) = Q_0$ (4)

where Q is the amount of substance at time t, K is a positive real number and Q_0 is the initial amount of the substance at time $t = t_0$.

In equation (3), the negative sign in the R.H.S. is taken because the mass of the substance is decreasing with time and so the derivative $\frac{dQ}{dt}$ must be negative.

In this paper, Sawi transform is used for finding the solution of population growth and decay problems.

II. LINEARITY PROPERTY OF SAWI TRANSFORMS

If $S\{F(t)\} = R_1(v)$ and $S\{G(t)\} = R_2(v)$ then

$$S\{aF(t) + bG(t)\} = aS\{F(t)\} + bS\{G(t)\}$$

$\Rightarrow S\{aF(t) + bG(t)\} = aR_1(v) + bR_2(v)$, where a, b are arbitrary constants.

III. SAWI TRANSFORM OF SOME USEFUL FUNCTIONS [64]:

Table-1

S.N.	$F(t)$	$S\{F(t)\} = R(v)$
1.	1	$\frac{1}{v}$
2.	t	1
3.	t^2	$2!\,v$
4.	$t^n, n \in N$	$n!\,v^{n-1}$
5.	$t^n, n > -1$	$\Gamma(n+1)v^{n-1}$
6.	e^{at}	$\frac{1}{v(1-av)}$
7.	$sinat$	$\frac{a}{1+a^2v^2}$
8.	$cosat$	$\frac{1}{v(1+a^2v^2)}$
9.	$sinhat$	$\frac{a}{1-a^2v^2}$
10.	$coshat$	$\frac{1}{v(1-a^2v^2)}$

IV. INVERSE SAWI TRANSFORM

If $S\{F(t)\} = R(v)$ then $F(t)$ is called the inverse Sawi transform of $R(v)$ and mathematically it is defined as $F(t) = S^{-1}\{R(v)\}$, where the operator S^{-1} is called the inverse Sawi transform operator.

V. INVERSE SAWI TRANSFORM OF SOME USEFUL FUNCTIONS

Table-2

S.N.	$R(v)$	$F(t) = S^{-1}\{R(v)\}$
1.	$\frac{1}{v}$	1
2.	1	t
3.	v	$\frac{t^2}{2!}$
4.	$v^{n-1}, n \in N$	$\frac{t^n}{n!}$
5.	$v^{n-1}, n > -1$	$\frac{t^n}{\Gamma(n+1)}$
6.	$\frac{1}{v(1-av)}$	e^{at}
7.	$\frac{1}{1+a^2v^2}$	$\frac{sinat}{a}$
8.	$\frac{1}{v(1+a^2v^2)}$	$cosat$
9.	$\frac{1}{1-a^2v^2}$	$\frac{sinhat}{a}$
10.	$\frac{1}{v(1-a^2v^2)}$	$coshat$

VI. SAWI TRANSFORM OF THE DERIVATIVES OF THE FUNCTION $F(t)$ [64]:

If $S\{F(t)\} = R(v)$ then

a) $S\{F'(t)\} = \frac{R(v)}{v} - \frac{F(0)}{v^2}$

b) $S\{F''(t)\} = \frac{R(v)}{v^2} - \frac{F(0)}{v^3} - \frac{F'(0)}{v^2}$

c) $S\{F^{(n)}(t)\} = \frac{R(v)}{v^n} - \frac{F(0)}{v^{n+1}} - \frac{F'(0)}{v^n} - \cdots \ldots - \frac{F^{(n-1)}(0)}{v^2}$.

VII. SAWI TRANSFORM FOR HANDLING POPULATION GROWTH PROBLEM

In this section, we present Sawi transform for population growth problem which is mathematically given by (1) and (2).

Taking Sawi transform on both sides of (1), we have

$$S\left\{\frac{dQ}{dt}\right\} = KS\{Q(t)\} \quad (5)$$

Now applying the property, Sawi transform of derivative of function, on (5), we have

$$\frac{1}{v}S\{Q(t)\} - \frac{Q(0)}{v^2} = KS\{Q(t)\} \quad (6)$$

Using (2) in (6) and on simplification, we have

$$\left(\frac{1}{v} - K\right) S\{Q(t)\} = \frac{Q_0}{v^2}$$

$$\Rightarrow S\{Q(t)\} = \frac{Q_0}{v(1-Kv)} \quad (7)$$

Operating inverse Sawi transform on both sides of (7), we have

$$Q(t) = S^{-1}\left\{\frac{Q_0}{v(1-Kv)}\right\}$$

$$\Rightarrow Q(t) = Q_0 S^{-1}\left\{\frac{1}{v(1-Kv)}\right\}$$

$$\Rightarrow Q(t) = Q_0 e^{Kt} \quad (8)$$

which is the required amount of the population at time t.

VIII. SAWI TRANSFORM FOR HANDLING DECAY PROBLEM

In this section, we present Sawi transform for decay problem which is mathematically expressed in terms of (3) and (4).

Applying the Sawi transform on both sides of (3), we have

$$S\left\{\frac{dQ}{dt}\right\} = -KS\{Q(t)\} \quad (9)$$

Now applying the property, Sawi transform of derivative of function, on (9), we have

$$\frac{1}{v}S\{Q(t)\} - \frac{Q(0)}{v^2} = -KS\{Q(t)\} \quad (10)$$

Using (4) in (10) and on simplification, we have

$$\left(\frac{1}{v} + K\right) S\{Q(t)\} = \frac{Q_0}{v^2}$$

$$\Rightarrow S\{Q(t)\} = \frac{Q_0}{v(1+Kv)} \quad (11)$$

Operating inverse Sawi transform on both sides of (11), we have

$$Q(t) = S^{-1}\left\{\frac{Q_0}{v(1+Kv)}\right\}$$

$$\Rightarrow Q(t) = Q_0 S^{-1}\left\{\frac{1}{v(1+Kv)}\right\}$$

$$\Rightarrow Q(t) = Q_0 e^{-Kt} \qquad (12)$$

which is the required amount of substance at time t.

IX. APPLICATIONS

In this section, some applications are given in order to demonstrate the effectiveness of Sawi transform for population growth and decay problems.

Application: 9.1 The population of a city grows at a rate proportional to the number of people presently living in the city. If after four years, the population has tripled, and after five years the population is 50,000, estimate the number of people initially living in the city.

This problem can be written in mathematical form as:

$$\frac{dQ(t)}{dt} = KQ(t) \qquad (13)$$

where Q denote the number of people living in the city at any time t and K is the constant of proportionality. Consider Q_0 is the number of people initially living in the city at $t = 0$.

Applying the Sawi transform on both sides of (13), we have

$$S\left\{\frac{dQ}{dt}\right\} = KS\{Q(t)\} \qquad (14)$$

Now applying the property, Sawi transform of derivative of function, on (14), we have

$$\frac{1}{v}S\{Q(t)\} - \frac{Q(0)}{v^2} = KS\{Q(t)\} \qquad (15)$$

Since at $t = 0, Q = Q_0$, so using this in (15), we have

$$\left(\frac{1}{v} - K\right) S\{Q(t)\} = \frac{Q_0}{v^2}$$

$$\Rightarrow S\{Q(t)\} = \frac{Q_0}{v(1-Kv)} \qquad (16)$$

Operating inverse Sawi transform on both sides of (16), we have

$$Q(t) = S^{-1}\left\{\frac{Q_0}{v(1-Kv)}\right\}$$

$$\Rightarrow Q(t) = Q_0 S^{-1}\left\{\frac{1}{v(1-Kv)}\right\}$$

$$\Rightarrow Q(t) = Q_0 e^{Kt} \qquad (17)$$

Now at $t = 4$, $Q = 3Q_0$, so using this in (17), we have

$$3Q_0 = Q_0 e^{4K}$$

$$\Rightarrow e^{4K} = 3$$

$$\Rightarrow K = \frac{1}{4}log_e 3 = 0.275 \qquad (18)$$

Now using the condition at $t = 5, Q = 50{,}000$, in (17), we have

$$50{,}000 = Q_0 e^{5K} \qquad (19)$$

Putting the value of K from (18) in (19), we have

$$50{,}000 = Q_0 e^{5\times 0.275}$$

$$\Rightarrow 50{,}000 = 3.955Q_0$$

$$\Rightarrow Q_0 \simeq 12642 \qquad (20)$$

which are the required number of people initially living in the city.

Application: 9.2 A radioactive substance is known to decay at a rate proportional to the amount present. If initially there is 100 milligrams of the radioactive substance present and after six hours it is observed that the radioactive substance has lost 30 percent of its original mass, find the half life of the radioactive substance.

This problem can be written in mathematical form as:

$$\frac{dQ(t)}{dt} = -KQ(t) \qquad (21)$$

where Q denote the amount of radioactive substance at time t and K is the constant of proportionality. Consider Q_0 is the initial amount of the radioactive substance at time $t = 0$.

Applying the Sawi transform on both sides of (21), we have

$$S\left\{\frac{dQ}{dt}\right\} = -KS\{Q(t)\} \qquad (22)$$

Now applying the property, Sawi transform of derivative of function, on (22), we have

$$\frac{1}{v}S\{Q(t)\} - \frac{Q(0)}{v^2} = -KS\{Q(t)\} \qquad (23)$$

Since at $t = 0, Q = Q_0 = 100$, so using this in (23), we have

$$\frac{1}{v}S\{Q(t)\} - \frac{100}{v^2} = -KS\{Q(t)\}$$

$$\Rightarrow \left(\frac{1}{v} + K\right) S\{Q(t)\} = \frac{100}{v^2}$$

$$\Rightarrow S\{Q(t)\} = \frac{100}{v(1+Kv)} \qquad (24)$$

Operating inverse Sawi transform on both sides of (24), we have

$$Q(t) = S^{-1}\left\{\frac{100}{v(1+Kv)}\right\}$$

$$= 100S^{-1}\left\{\frac{1}{v(1+Kv)}\right\}$$

$$\Rightarrow Q(t) = 100e^{-Kt} \qquad (25)$$

Now at $t = 6$, the radioactive substance has lost 30 percent of its original mass 100 mg so $Q = 100 - 30 = 70$, using this in (25), we have

$$70 = 100e^{-6K}$$

$$\Rightarrow e^{-6K} = 0.70$$

$$\Rightarrow K = -\frac{1}{6} log_e 0.70 = 0.059 \qquad (26)$$

We required t when $Q = \frac{Q_0}{2} = \frac{100}{2} = 50$ so from (25), we have

$$50 = 100e^{-Kt} \qquad (27)$$

Putting the value of K from (26) in (27), we have

$$50 = 100e^{-0.059t}$$

$$\Rightarrow e^{-0.059t} = 0.50$$

$$\Rightarrow t = -\frac{1}{0.059} log_e 0.50$$

$$\Rightarrow t = 11.75 \text{ hours} \qquad (28)$$

which is the required half-time of the radioactive substance.

X. CONCLUSION

In this paper, we have successfully discussed the Sawi transform for population growth and decay problems. The given numerical applications in application section show the importance of Sawi transform for population growth and decay problems. Results show that Sawi transform is very useful integral transform for handling the population growth and decay problems. The scheme defined in this paper can be applied for finding the solution of continuous compound interest problems, heat conduction problems, vibrating beam problems, mixture problems and electric circuit problems.

CONFLICT OF INTEREST

The authors confirm that this article contents have no conflict of interest.

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