Determination of Displacement Functions of Thick Anisotropic Rectangular Plate by Direct Integration of Governing Equation

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Abstract: **- Total potential energy equation of thick anisotropic rectangular plate was differentiated with various displacement functions w, ϕ_x and ϕ_y. The results yielded the governing equations of thick anisotropic rectangular plate as shown in equation (1). The governing equation (1a) of displacement w was then minimized by adding the similar terms together to yield equation (9). The displacement functions of equation (9) were split by substituting w for$w_x.w_y$, ϕ_x for $\phi_{xx}.\phi_{xy}$ and ϕ_y for $\phi_{yx}.\phi_{yy}$ respectively. The split equation (14) was solved to obtain equations (33), (34), (35), (36), (37) and (38) respectively. These various differential and integral functions were then integrated to obtain the general displacement function solutions that many authors have been assuming when analyzing thick, thin, isotropic, orthotropic and anisotropic plates. These general displacement functions were determined in equations (40), (42), (44), (46), (49), (50), (51), (52) and (53) respectively. From the general displacement function we obtained the exact displacement function for SSSS rectangular plate as presented on Table 1. These exact displacement functions for SSSS rectangular plate can be used in analyzing thick anisotropic plate at any point.**

Key Words:-**Potential; Energy; Anisotropic; Thick; Plate; Displacement and Governing Equation.**

I. INTRODUCTION

Researchers channeled more attention to composite materials in the middle of the twentieth century. It was then recommended as the promising class of engineering materials because of its prospective quality for modern technology. Broadly speaking, any material that is made up of two or more components with different properties and distinct boundaries between the components can be referred to as a composite material (Vasiliev and Morozov, 2013).

The inventions of these high performance reinforced composite fiber materials have attracted more interest to the solution of plates with anisotropic properties. There are cases in which an anisotropic material must be assumed if we wish to bring the theory of plate into agreement with experiment (Timoshenko and Woinowsky, 1959). Unlike isotropic that assume material properties at a point to be the same in all directions however, certain materials display direction-dependent properties; consequently, these materials are referred to as anisotropic material (Ventsel and Krauthammer, 2001). These materials show different resistance to their mechanical action in different direction. Anisotropic plates are those plates in which difference between the flexural rigidities for different directions was created artificially examples are corrugated plates, plates made with aviation plywood, texolite etc (Lekhnitskiy, 1968).

Methods like Fourier, Navier, Galerkin, Ritz etc used approximate or readymade solution and assumed displacement function to solve the governing equations of thick anisotropic rectangular plate. However none of these scholars actually tried to verify the validity of those assumed displacement function and the approximate solutions. This work is focused on verifying how true those assumed solution and displacement function are by directly integrating the governing equation to obtain its solutions as well as the displacement functions from its governing equation.

II. METHODOLOGY

The governing equations of thick anisotropic rectangular plate are as follows:

$$\frac{d\pi}{dw} = \int_0^1 \int_0^1 \{c_{11}[\frac{d^4w}{dx^4} - g_2\frac{d^3\phi_x}{dx^3}] + c_{12}[\frac{d^4w}{dx^2dy^2} - \frac{g_2}{2}\frac{d^3\phi_y}{dx^2dy} - \frac{g_2}{2}\frac{d^3\phi_x}{dx\,dy^2}] + c_{13}[2\frac{d^4w}{dx^3dy} - \frac{3g_2}{2}\frac{d^3\phi_x}{dx^2dy} - \frac{g_2}{2}\frac{d^3\phi_y}{dx^3}] + c_{21}[\frac{d^4w}{dx^2dy^2} - \frac{g_2}{2}\frac{d^3\phi_y}{dx^2dy} - \frac{g_2}{2}\frac{d^3\phi_x}{dx\,dy^2}] + c_{22}[\frac{d^4w}{dy^4} - g_2\frac{d^3\phi_y}{dy^3}] + c_{23}[2\frac{d^4w}{dxdy^3} - \frac{3g_2}{2}\frac{d^3\phi_y}{dxdy^2} - \frac{g_2}{2}\frac{d^3\phi_x}{dy^3}] + c_{31}[2\frac{d^4w}{dx^3dy} - \frac{3g_2}{2}\frac{d^3\phi_x}{dx^2dy} - \frac{g_2}{2}\frac{d^3\phi_y}{dx^3}] + c_{32}[2\frac{d^4w}{dxdy^3} - \frac{3g_2}{2}\frac{d^3\phi_y}{dxdy^2} - \frac{g_2}{2}\frac{d^3\phi_x}{dy^3}] + c_{33}[4\frac{d^4w}{dx^2dy^2} - 2g_2\frac{d^3\phi_y}{dx^2dy} - 2g_2\frac{d^3\phi_x}{dx\,dy^2}] - \frac{q}{D_{11}}\}\,\mathrm{dxdy} = 0 \quad \text{1a}$$

$$\frac{d\pi}{d\phi_x} = \int_0^1 \int_0^1 \{c_{11}[-g_2\frac{d^3w}{dx^3} + g_3\frac{d^2\phi_x}{dx^2}] + c_{12}[-\frac{g_2}{2}\frac{d^3w}{dx\,dy^2} + \frac{g_3}{2}\frac{d^2\phi_y}{dxdy}] + c_{13}[-\frac{3g_2}{2}\frac{d^3w}{dx^2dy} + g_3\frac{d^2\phi_x}{dxdy} + \frac{g_3}{2}\frac{d^2\phi_y}{dx^2}] + c_{21}[-\frac{g_2}{2}\frac{d^3w}{dx\,dy^2} + \frac{g_3}{2}\frac{d^2\phi_y}{dxdy}] + c_{23}[-\frac{g_2}{2}\frac{d^3w}{dy^3} + \frac{g_3}{2}\frac{d^2\phi_y}{dy^2}] + c_{31}[-\frac{3g_2}{2}\frac{d^3w}{dx^2dy} + g_3\frac{d^2\phi_x}{dxdy} + \frac{g_3}{2}\frac{d^2\phi_y}{dx^2}] + c_{32}[-\frac{g_2}{2}\frac{d^3w}{dy^3} +$$

$$\frac{g_3}{2}\frac{d^2\phi_y}{dy^2}] + c_{33}[-2g_2 \frac{d^3w}{dxdy^2} + g_3 \frac{d^2\phi_y}{dxdy} + g_3 \frac{d^2\phi_x}{dy^2}] + c_{44}\alpha^2 g_4\phi_x + c_{45}\frac{\alpha^2 g_4}{2}\phi_y + c_{54}\frac{\alpha^2 g_4}{2}\phi_y \} \text{dxdy} = 0 \quad \text{1b}$$

$$\frac{d\pi}{d\phi_y} = \int_0^1\int_0^1 \{ c_{12}[-\frac{g_2}{2}\frac{d^3w}{dx^2dy} + \frac{g_3}{2}\frac{d^2\phi_x}{dxdy}] + c_{13}[-\frac{g_2}{2}\frac{d^3w}{dx^3} + \frac{g_3}{2}\frac{d^2\phi_x}{dx^2}] + c_{21}[-\frac{g_2}{2}\frac{d^3w}{dx^2dy} + \frac{g_3}{2}\frac{d^2\phi_x}{dxdy}]] + c_{22}[-g_2\frac{d^3w}{dy^3} + g_3\frac{d^2\phi_y}{dy^2}] + c_{23}[-\frac{3g_2}{2}\frac{d^3w}{dxdy^2} + g_3\frac{d^2\phi_y}{dxdy} + \frac{g_3}{2}\frac{d^2\phi_x}{dy^2}] + c_{31}[-\frac{g_2}{2}\frac{d^3w}{dx^3} + \frac{g_3}{2}\frac{d^2\phi_x}{dx^2}] + c_{32}[-\frac{3g_2}{2}\frac{d^3w}{dxdy^2} + g_3\frac{d^2\phi_y}{dxdy} + \frac{g_3}{2}\frac{d^2\phi_x}{dy^2}] + c_{33}[-2g_2\frac{d^3w}{dx^2dy} + g_3\frac{d^2\phi_x}{dxdy} + g_3\frac{d^2\phi_y}{dx^2}] + c_{45}\frac{\alpha^2 g_4}{2}\phi_x + c_{54}\frac{\alpha^2 g_4}{2}\phi_x + c_{55}\alpha^2 g_4\phi_y \} \text{dxdy} = 0 \quad \text{1c}$$

Where $c_{11} = \frac{D_{11}}{D_{11}}$ 2

$c_{12} = \frac{D_{12}}{D_{11}}$ 3

$c_{13} = \frac{D_{13}}{D_{11}}$ 4

$c_{ij} = \frac{D_{ij}}{D_{11}}$ 5

From equation (1) we observed that;

$c_{12} = c_{21} = 4c_{33}$ 6

$c_{13} = c_{31}$ 7

$c_{23} = c_{32}$ 8

Adding together the like terms of equation (6), (7) and (8) in equation (1) gives

$$\frac{d\pi}{dw} = \int_0^1\int_0^1 \{c_{11}[\frac{d^4w}{dx^4} - g_2\frac{d^3\phi_x}{dx^3}] + 3s_{12}[\frac{d^4w}{dx^2dy^2} - \frac{g_2}{2}\frac{d^3\phi_y}{dx^2dy} - \frac{g_2}{2}\frac{d^3\phi_x}{dxdy^2}] + 2s_{13}[2\frac{d^4w}{dx^3dy} - \frac{3g_2}{2}\frac{d^3\phi_x}{dx^2dy} - \frac{g_2}{2}\frac{d^3\phi_y}{dx^3}] + c_{22}[\frac{d^4w}{dy^4} - g_2\frac{d^3\phi_y}{dy^3}] + 2s_{23}[2\frac{d^4w}{dxdy^3} - \frac{3g_2}{2}\frac{d^3\phi_y}{dxdy^2} - \frac{g_2}{2}\frac{d^3\phi_x}{dy^3}] \} \text{dxdy} = \frac{q}{D_{11}} \quad 9$$

Where $3s_{12} = c_{12} + c_{21} + 4c_{33}$ 9a

$2s_{13} = c_{13} + c_{31}$ 9b

$2s_{23} = c_{23} + c_{32}$ 9c

Let x = aR 9d

Y = bQ 9e

Aspect ratio, $\alpha = \frac{b}{a}$ 9f

Substituting equations (10), (11) and (12) into equation (9) and rearranging it gives

$$\frac{d\pi}{dw} = \int_0^1\int_0^1 \{ \frac{d^4w}{dR^4}c_{11} + \frac{3}{\alpha^2}\frac{d^4w}{dR^2dQ^2}s_{12} + \frac{4}{\alpha}\frac{d^4w}{dR^3dQ}s_{13} + \frac{1}{\alpha^4}\frac{d^4w}{dQ^4}c_{22} + \frac{4}{\alpha^3}\frac{d^4w}{dRdQ^3}s_{23} - ag_2\frac{d^3\phi_x}{dR^3}c_{11} - \frac{3ag_2}{2\alpha^2}\frac{d^3\phi_x}{dRdQ^2}s_{12} - \frac{3ag_2}{\alpha}\frac{d^3\phi_x}{dR^2dQ}s_{13} - \frac{ag_2}{\alpha^3}\frac{d^3\phi_x}{dQ^3}s_{23} - \frac{3ag_2}{2\alpha}\frac{d^3\phi_y}{dR^2dR}s_{12} - ag_2\frac{d^3\phi_y}{dR^3}s_{13} - \frac{ag_2}{\alpha^3}\frac{d^3\phi_y}{dQ^3}c_{22} - \frac{3ag_2}{\alpha^2}\frac{d^3\phi_y}{dRdQ^2}s_{23} - \frac{qa^4}{D_{11}} \} \text{dRdQ} = 0 \quad 10$$

Let $w = w_x . w_x$ 11

$\phi_x = \phi_{xx} . \phi_{xy}$ 12

$\phi_y = \phi_{yx} . \phi_{yy}$ 13

Splitting the displacements w, $\phi_{x\ and}\ \phi_y$ by substituting equations (11), (12) and (13) into equation (10) gives.

$$\frac{d\pi}{dw} = \int_0^1\int_0^1 \{ w_y\frac{d^4w_x}{dR^4}c_{11} + \frac{3}{\alpha^2}\frac{d^2w_x}{dR^2}\frac{d^2w_y}{dQ^2}s_{12} + \frac{w_x}{\alpha^4}\frac{d^4w_y}{dQ^4}c_{22} + \frac{4}{\alpha}\frac{d^3w_x}{dR^3}\frac{dw_y}{dQ}s_{13} + \frac{4}{\alpha^3}\frac{dw_x}{dR}\frac{d^3w_y}{dQ^3}s_{23} - ag_2\phi_{xy}\frac{d^3\phi_{xx}}{dR^3}c_{11} - \frac{3ag_2}{2\alpha^2}\frac{d\phi_{xx}}{dR}\frac{d^2\phi_{xy}}{dQ^2}s_{12} - \frac{3ag_2}{\alpha}\frac{d^2\phi_{xx}}{dR^2}\frac{d\phi_{xy}}{dQ}s_{13} - \frac{3ag_2}{2\alpha}\frac{d^2\phi_{yx}}{dR^2}\frac{d\phi_{yy}}{dQ}s_{12} - \frac{3ag_2}{\alpha^2}\frac{d\phi_{yx}}{dR}\frac{d^2\phi_{yy}}{dQ^2}s_{23} - \frac{ag_2}{\alpha^3}\phi_{xx}\frac{d^3\phi_{xy}}{dQ^3}s_{23} - ag_2\phi_{yy}\frac{d^3\phi_{yx}}{dR^3}s_{13} - \frac{ag_2}{\alpha^3}\phi_{yx}\frac{d^3\phi_{yy}}{dQ^3}c_{22} - \frac{qa^4}{D_{11}} \} \text{dRdQ} = 0 \quad 14$$

Let $1 = n_1 + n_2 + n_3 - n_4 - n_5$ 15

Substituting equation (15) into equation (14) gives;

$$\int_0^1\int_0^1 \{ w_y\frac{d^4w_x}{dR^4}c_{11} + \frac{3}{\alpha^2}\frac{d^2w_x}{dR^2}\frac{d^2w_y}{dQ^2}s_{12} + \frac{w_x}{\alpha^4}\frac{d^4w_y}{dQ^4}c_{22} + \frac{4}{\alpha}\frac{d^3w_x}{dR^3}\frac{dw_y}{dQ}s_{13} + \frac{4}{\alpha^3}\frac{dw_x}{dR}\frac{d^3w_y}{dQ^3}s_{23} - ag_2\phi_{xy}\frac{d^3\phi_{xx}}{dR^3}c_{11} - \frac{3ag_2}{2\alpha^2}\frac{d\phi_{xx}}{dR}\frac{d^2\phi_{xy}}{dQ^2}s_{12} - \frac{3ag_2}{\alpha}\frac{d^2\phi_{xx}}{dR^2}\frac{d\phi_{xy}}{dQ}s_{13} - \frac{3ag_2}{2\alpha}\frac{d^2\phi_{yx}}{dR^2}\frac{d\phi_{yy}}{dQ}s_{12} - \frac{3ag_2}{\alpha^2}\frac{d\phi_{yx}}{dR}\frac{d^2\phi_{yy}}{dQ^2}s_{23} - \frac{ag_2}{\alpha^3}\phi_{xx}\frac{d^3\phi_{xy}}{dQ^3}s_{23} - ag_2\phi_{yy}\frac{d^3\phi_{yx}}{dR^3}s_{13} - \frac{ag_2}{\alpha^3}\phi_{yx}\frac{d^3\phi_{yy}}{dQ^3}c_{22} - \frac{qa^4}{D}[n_1 + n_2 + n_3 - n_4 - n_5] \} \text{dRdQ} = 0$$

Which implies that;

$$\int_0^1\int_0^1 \{ (w_y\frac{d^4w_x}{dR^4}c_{11} - \frac{qa^4}{D}n_1) + \left(\frac{3}{\alpha^2}\frac{d^2w_x}{dR^2}\frac{d^2w_y}{dQ^2}s_{12} - \frac{qa^4}{D}n_2\right) + \left(\frac{w_x}{\alpha^4}\frac{d^4w_y}{dQ^4}c_{22} - \frac{qa^4}{D}n_3\right) + \left(\frac{4}{\alpha}\frac{d^3w_x}{dR^3}\frac{dw_y}{dQ}s_{13} + \frac{4}{\alpha^3}\frac{dw_x}{dR}\frac{d^3w_y}{dQ^3}s_{23}\right) - (ag_2\phi_{xy}\frac{d^3\phi_{xx}}{dR^3}c_{11} - \frac{qa^4}{D}n_4) - (\frac{3ag_2}{2\alpha^2}\frac{d\phi_{xx}}{dR}\frac{d^2\phi_{xy}}{dQ^2}s_{12} + \frac{3ag_2}{2\alpha}\frac{d^2\phi_{yx}}{dR^2}\frac{d\phi_{yy}}{dQ}s_{12}) - (\frac{3ag_2}{\alpha}\frac{d^2\phi_{xx}}{dR^2}\frac{d\phi_{xy}}{dQ}s_{13} + ag_2\phi_{yy}\frac{d^3\phi_{yx}}{dR^3}s_{13}) + (\frac{ag_2}{\alpha^3}\phi_{xx}\frac{d^3\phi_{xy}}{dQ^3}s_{23} + \frac{3ag_2}{\alpha^2}\frac{d\phi_{yx}}{dR}\frac{d^2\phi_{yy}}{dQ^2}s_{23}) - (\frac{ag_2}{\alpha^3}\phi_{yx}\frac{d^3\phi_{yy}}{dQ^3}c_{22} - \frac{qa^4}{D}n_5) \} \text{dRdQ} = 0 \quad 16$$

Each bracket in equation (16) must be equal to zero for its equation to be true;

$$\int_0^1\int_0^1(w_y \frac{d^4w_x}{dR^4} c_{11} - \frac{qa^4}{D} n_1) \text{ dRdQ } = 0 \qquad 17$$

$$\int_0^1\int_0^1(\frac{3}{\alpha^2}\frac{d^2w_x}{dR^2}\frac{d^2w_y}{dQ^2} s_{12} - \frac{qa^4}{D} n_2) \text{ dRdQ} = 0 \qquad 18$$

$$\int_0^1\int_0^1(\frac{w_x}{\alpha^4}\frac{d^4w_y}{dQ^4} c_{22} - \frac{qa^4}{D} n_3) \text{ dRdQ} = 0 \qquad 19$$

$$\int_0^1\int_0^1(ag_2\phi_{xy}\frac{d^3\phi_{xx}}{dR^3} c_{11} - \frac{qa^4}{D} n_4) \text{ dRdQ} = 0 \qquad 20$$

$$\int_0^1\int_0^1(\frac{ag_2}{\alpha^3}\phi_{yx}\frac{d^3\phi_{yy}}{dQ^3} c_{22} - \frac{qa^4}{D} n_5) \text{ dRdQ} = 0 \qquad 21$$

$$\int_0^1\int_0^1(\frac{3ag_2}{2\alpha^2}\frac{d\phi_{xx}}{dR}\frac{d^2\phi_{xy}}{dQ^2} s_{12} + \frac{3ag_2}{2\alpha}\frac{d^2\phi_{yx}}{dR^2}\frac{d\phi_{yy}}{dQ} s_{12}) \text{ dRdQ} = 0 \qquad 22$$

Integrating equation (17) in its closed domain with respect to Q gives;

$$\int_0^1(w_3 \frac{d^4w_x}{dR^4} c_{11} - \frac{qa^4}{D} n_1) \text{ dR} = 0 \qquad 23$$

Integrating equation (19) in its closed domain with respect to R gives;

$$\int_0^1(\frac{w_1}{\alpha^4}\frac{d^4w_y}{dQ^4} c_{22} - \frac{qa^4}{D} n_3) \text{ dQ} = 0 \qquad 24$$

Integrating equation (20) in its closed domain with respect to Q gives;

$$\int_0^1(ag_2\phi_3\frac{d^3\phi_{xx}}{dR^3} c_{11} - \frac{qa^4}{D} n_4) \text{ dR} = 0 \qquad 25$$

Integrating equation (21) in its closed domain with respect to R gives;

$$\int_0^1(\frac{ag_2}{\alpha^3}\phi_1\frac{d^3\phi_{yy}}{dQ^3} c_{22} - \frac{qa^4}{D} n_5) \text{ dQ} = 0 \qquad 26$$

Integrating equation (22) in its closed domain with respect to R gives;

$$\int_0^1(\frac{3ag_2}{2\alpha^2}\phi_4\frac{d^2\phi_{xy}}{dQ^2} s_{12} + \frac{3ag_2}{2\alpha}\phi_5\frac{d\phi_{yy}}{dQ} s_{12}) \text{ dQ} = 0 \qquad 27$$

Integrating equation (22) in its closed domain with respect to Q gives;

$$\int_0^1(\frac{3ag_2}{2\alpha^2}\frac{d\phi_{xx}}{dR}\phi_6 s_{12} + \frac{3ag_2}{2\alpha}\frac{d^2\phi_{yx}}{dR^2}\phi_7 s_{12}) \text{ dQ} = 0 \qquad 28$$

Where

$w_1 = \int_0^1 (w_x)\text{dR}$; $w_3 = \int_0^1 (w_y)\text{dQ}$;

$\phi_1 = \int_0^1 (\phi_{yx})\text{dR}$; $\phi_3 = \int_0^1 (\phi_{xy})\text{dQ}$;

$\phi_4 = \int_0^1 (\frac{d\phi_{XX}}{dR})\text{dR}$; $\phi_5 = \int_0^1 (\frac{d^2\phi_{yx}}{dR^2})\text{dR}$;

$\phi_6 = \int_0^1 (\frac{d^2\phi_{xy}}{dQ^2})\text{dQ}$; $\phi_7 = \int_0^1 (\frac{d\phi_{yy}}{dQ})\text{Dq}$

For equations (23), (24), (25), (26), (27) and (28) to be true, their integrands must be zero. That is;

$$w_3 \frac{d^4w_x}{dR^4} c_{11} - \frac{qa^4}{D} n_1 = 0 \qquad 29$$

$$\frac{w_1}{\alpha^4}\frac{d^4w_y}{dQ^4} c_{22} - \frac{qa^4}{D} n_3 = 0 \qquad 30$$

$$ag_2\phi_3\frac{d^3\phi_{xx}}{dR^3} c_{11} - \frac{qa^4}{D} n_4 = 0 \qquad 31$$

$$\frac{ag_2}{\alpha^3}\phi_1\frac{d^3\phi_{yy}}{dQ^3} c_{22} - \frac{qa^4}{D} n_5 = 0 \qquad 32$$

$$\frac{3ag_2}{2\alpha^2}\phi_4\frac{d^2\phi_{xy}}{dQ^2} s_{12} + \frac{3ag_2}{2\alpha}\phi_5\frac{d\phi_{yy}}{dQ} s_{12} = 0 \qquad 33$$

$$\frac{3ag_2}{2\alpha^2}\frac{d\phi_{xx}}{dR}\phi_6 s_{12} + \frac{3ag_2}{2\alpha}\frac{d^2\phi_{yx}}{dR^2}\phi_7 s_{12} = 0 \qquad 34$$

Equations (29), (30), (31), (32), (33) and (34) can be rearranged as follows;

$$\frac{d^4w_x}{dR^4} = \frac{qa^4}{D}(\frac{n_1}{w_3c_{11}}) \qquad 35$$

$$\frac{d^4w_y}{dQ^4} = \frac{qa^4}{D}(\frac{n_3\alpha^4}{w_1c_{22}}) \qquad 36$$

$$\frac{d^3\phi_{xx}}{dR^3} = \frac{qa^3}{D}(\frac{n_4}{g_2\phi_3c_{11}}) \qquad 37$$

$$\frac{d^3\phi_{yy}}{dQ^3} = \frac{qa^3}{D}(\frac{n_5\alpha^3}{g_2\phi_1c_{22}}) \qquad 38$$

Integrating equation (35) four times with respect to R gives;

$$\int(\frac{d^4w_x}{dR^4})\text{dR} = (\frac{n_1}{w_3c_{11}})\int(\frac{qa^4}{D})\text{dR}$$

$$\int(\frac{d^3w_x}{dR^3})\text{dR} = (\frac{n_1}{w_3c_{11}})\int(\frac{qa^4}{D}\text{R} + a_3)\text{dR}$$

$$\int(\frac{d^2w_x}{dR^2})\text{dR} = (\frac{n_1}{w_3c_{11}})\int(\frac{qa^4}{2D}R^2 + a_3R + a_2)\text{dR}$$

$$\int(\frac{dw_x}{dR})\text{dR} = (\frac{n_1}{w_3c_{11}})\int(\frac{qa^4}{6D}R^3 + R^2\frac{a_2}{2} + a_1)\text{dR}$$

$$w_x = (\frac{n_1}{w_3c_{11}})\frac{qa^4}{24D}R^4 + R^3\frac{a_3}{6} + R^2\frac{a_2}{2} + \text{R}a_1 + a_0 \qquad 39$$

Equation (39) can be cast into matrix as given;

$$w_x = [\,1 \;\; \text{R} \;\; R^2R^3R^4] \begin{bmatrix} a_0 \\ a_1 \\ (\frac{a_2}{2}) \\ (\frac{a_3}{6}) \\ (\frac{n_1}{w_3c_{11}})\frac{qa^4}{24D} \end{bmatrix} = [h_x\,][A_{x1}] \qquad 40$$

Similarly integrating equation (36) four times with respect to Q gives

$$w_y = (\frac{n_3\alpha^4}{w_1c_{22}})\frac{qa^4}{24D}Q^4 + Q^3\frac{b_3}{6} + Q^2\frac{b_2}{2} + Qb_1 + b_0 \quad 41$$

Casting equation (41) into matrix gives

$$w_y = [1 \ Q \ \ Q^2Q^3Q^4]\begin{bmatrix} b_0 \\ b_1 \\ (\frac{b_2}{2}) \\ (\frac{b_2}{6}) \\ (\frac{n_3\alpha^4}{w_1c_{22}})\frac{qa^4}{24D} \end{bmatrix} = [h_y][A_{y1}] \quad 42$$

Similarly integrating equation (37) four times with respect to R gives

$$\phi_{xx} = (\frac{n_4}{g_2\phi_3c_{11}})\frac{qa^3}{6D}R^3 + R^2\frac{a_6}{2} + Ra_5 + a_4 \quad 43$$

Casting equation (43) into matrix gives;

$$\phi_{xx} = [1 \ R \ \ R^2R^3]\begin{bmatrix} a_4 \\ a_5 \\ (\frac{a_6}{2}) \\ (\frac{n_4}{g_2\phi_3c_{11}})\frac{qa^3}{6D} \end{bmatrix} = [\frac{dh_x}{dR}][A_{x2}] \quad 44$$

Similarly integrating equation (38) four times with respect to Q gives

$$\phi_{yy} = (\frac{n_5\alpha^3}{g_2\phi_1c_{22}})\frac{qa^3}{6D}Q^3 + Q^2\frac{b_6}{2} + Qb_5 + b_4 \quad 45$$

III. RESULTS

Casting equation (45) into matrix gives;

$$\phi_{yy} = [1 \ Q \ \ Q^2Q^3]\begin{bmatrix} b_4 \\ b_5 \\ (\frac{b_6}{2}) \\ (\frac{n_5\alpha^3}{g_2\phi_1c_{22}})\frac{qa^3}{6D} \end{bmatrix} = [\frac{dh_y}{dQ}][A_{y2}] \quad 46$$

Equation (33) and (34) can be solved to give;

$$\phi_{xy} = -\frac{\phi_5}{\phi_4}\alpha.\int(\phi_{yy})\,dQ \quad 47$$

$$\phi_{yx} = -\frac{\phi_6}{\alpha\phi_7}.\int(\phi_{xx})\,dR \quad 48$$

Substituting equation (44) into equation (48) gives;

$$\phi_{yx} = [1 \ R \ \ R^2R^3R^4]\begin{bmatrix} a_7 \\ a_8 \\ (\frac{a_9}{2}) \\ (\frac{a_{10}}{6}) \\ (\frac{a_{11}}{24}) \end{bmatrix} = [h_x][A_{x3}] \quad 49$$

Substituting equation (46) into equation (47) gives;

$$\phi_{xy} = [1 \ Q \ \ Q^2Q^3Q^4]\begin{bmatrix} b_7 \\ b_8 \\ (\frac{b_9}{2}) \\ (\frac{b_{10}}{6}) \\ (\frac{b_{11}}{24}) \end{bmatrix} = [h_y][A_{y3}] \quad 50$$

Substituting equations (40) and (42) into equation (11) gives;

$$W = [h_x][A_{x1}] \ x \ [h_y][A_{y1}] = A_1h \quad 51$$

Substituting equations (44) and (50) into equation (12) gives;

$$\phi_x = [\frac{dh_x}{dR}][A_{x2}] \ x \ [h_y][A_{y3}] = A_2\frac{dh}{dR} \quad 52$$

Substituting equations (49) and (46) into equation (13) gives;

$$\phi_y = [h_x][A_{x3}] \ x \ [\frac{dh_y}{dQ}][A_{y2}] = A_3\frac{dh}{dQ} \quad 53$$

Table 4.1 Displacement functions for SSSS rectangular plate

Displacement function		**BOUNDARY CONDITION** **SSSS Rectangular Plate**	**Non-dimensional values of displacements at various points**	**Points**
W	A_1h	$A_1(R-2R^3+R^4)(Q-2Q^3+Q^4)$	$0.09765625A_1$	$\frac{a}{2},\frac{b}{2}$
$\emptyset_x$	$A_2.\frac{dh}{dR}$	$A_2(1-6R^2+4R^3)(Q-2Q^3+Q^4)$	$0.3125A_2$	$0,\frac{b}{2}$
$\emptyset_y$	$A_3.\frac{dh}{dQ}$	$A_3(R-2R^3+R^4)(1-6Q^2+4Q^3)$	$0.3125A_3$	$\frac{a}{2}$, 0
$\emptyset_x^I$	$A_2.\frac{d^2h}{dR^2}$	$A_2(R^2-R)(Q-2Q^3+Q^4).12$	$-0.9375A_2$	$\frac{a}{2},\frac{b}{2}$
$\emptyset_y^I$	$A_3.\frac{d^2h}{dQ^2}$	$A_3(R-2R^3+R^4)(Q^2-Q).12$	$-0.9375A_2$	$\frac{a}{2},\frac{b}{2}$
W_{xy}^{II}	$A_1.\frac{d^2w}{dRdQ}$	$A_1(1-6R^2+4R^3)(1-6Q^2+4Q^3)$	$1.A_1$	a, b, & 0, 0

IV. CONCLUSIONS

The results of equation (40), (42), (44), (46), (47), (48), (49), (50), (51), (52), (53) and Table 1 agreed with the approximate solutions and displacement functions that were assumed by past scholars like Reddy (2004), Reddy and Arciniega(2004), Joshan et al (2017) etc. Hence the results authenticates the assumptions made by past scholars and can always be used for verification of those assumptions. This exact displacement function can be used to analyze SSSS anisotropic plate at any point on the rectangular plate. Also, the solution is simpler and easier to apply when analyzing thick anisotropic plate.

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