Method of Taylor's Series for the Solution of Non-Linear First Kind Volterra Integral Equations

Sudhanshu Aggarwal[1], Swarg Deep Sharma[2], Anjana Rani Gupta[3]

[1]*Assistant Professor, Department of Mathematics, National P.G. College, Barhalganj, Gorakhpur-273402, U.P., India,*
[2]*Assistant Professor, Department of Mathematics, Nand Lal Singh College Jaitpur Daudpur Constituent of Jai Prakash University Chhapra-841205, Bihar, India*
[3]*Professor, Department of Mathematics, Noida Institute of Engineering & Technology, Greater Noida-201306, U.P., India*

Abstract: **Integral equations have different applications such as determination of potentials, system identification, spectroscopy and seismic travel time. In this paper, authors have solved non-linear first kind Volterra integral equations (V.I.E.) using Taylor series method. Authors have been considered two numerical examples for explaining the complete methodology. Results of numerical examples show that Taylor series method is very useful and effective numerical method for handling the problem of obtaining the primitives of non-linear first kind V.I.E.**

Keywords: **Taylor series method; Volterra integral equation; Power series.**

AMS Subject Classification 2010**: 45D05, 45GXX, 35C10**

I. INTRODUCTION

With the remarkable advancement in different field of engineering, science, and technology, today more than ever before, the study of integral equations has become essential. For, to have an exhaustive understanding of subjects like fluid dynamics, numerical analysis, waves and electromagnetic, chemistry, physics, statistics, mechanics, heat transfer, chemical science, mathematical biology, aerodynamics, electricity the knowledge of determining the solution to integral equations is absolutely necessary. These integral equations may be linear or nonlinear. Finding and interpreting the solutions of these integral equations is therefore a central part of applied mathematics and a thorough understanding of integral equations is essential for any scholars. Aggarwal with others [1-5] used different integral transformations for obtaining the solutions of V.I.E. of second kind. The primitives of first kind V.I.E. were obtained by Aggarwal et al. [6-11] by applying Laplace; Kamal; Mahgoub; Aboodh; Elzaki; Shehu integral transformations on them. Aggarwal and others scholars [12-18] determined the exact solution of famous problem of mechanics (Abel's problem) by applying Laplace; Kamal; Mohand; Aboodh; Sumudu; Shehu; Sadik integral transformations on it. This problem was a special case of V.I.E.

The goal of this paper is to determine the solutions of non-linear first kind V.I.E. by applying Taylor series method on them.

II. POWER SERIES (TAYLOR SERIES) OF FREQUENTLY USED FUNCTIONS IN ENGINEERING AND MATHEMATICAL SCIENCE

$$e^{\tau} = \left[1 + \tau + \frac{\tau^2}{2!} + \frac{\tau^3}{3!} + \frac{\tau^4}{4!} + \frac{\tau^5}{5!} + \cdots \dots \dots \dots \dots\right]$$

$$e^{-\tau} = \left[1 - \tau + \frac{\tau^2}{2!} - \frac{\tau^3}{3!} + \frac{\tau^4}{4!} - \frac{\tau^5}{5!} + \cdots \dots \dots \dots\right]$$

$$e^{a\tau} = \left[1 + a\tau + \frac{(a\tau)^2}{2!} + \frac{(a\tau)^3}{3!} + \frac{(a\tau)^4}{4!} + \cdots\right]$$

$$e^{-a} = \left[1 - a\tau + \frac{(a\tau)^2}{2!} - \frac{(a\tau)^3}{3!} + \frac{(a\tau)^4}{4!} - \cdots \dots .\right]$$

$$sin\tau = \left[\tau - \frac{\tau^3}{3!} + \frac{\tau^5}{5!} - \frac{\tau^7}{7!} + \cdots \dots \dots \dots \dots \dots \dots \dots \dots \dots\right]$$

$$cos\tau = \left[1 - \frac{\tau^2}{2!} + \frac{\tau^4}{4!} - \frac{\tau^6}{6!} + \cdots \dots \dots \dots \dots \dots \dots \dots \dots .\right]$$

$$tan\tau = \left[\tau + \frac{\tau^3}{3} + \frac{2\tau^5}{15} + \cdots \dots \dots \dots \dots \dots \dots \dots \dots\right]$$

$$sinh\tau = \left[\tau + \frac{\tau^3}{3} + \frac{\tau^5}{5!} + \frac{\tau^7}{7!} + \cdots \dots \dots \dots \dots \dots \dots \dots \dots\right]$$

$$cosh\tau = \left[1 + \frac{\tau^2}{2!} + \frac{\tau^4}{4!} + \frac{\tau^6}{6!} + \cdots \dots \dots \dots \dots \dots \dots \dots \dots .\right]$$

$$sin^{-1}\tau = \left[\tau + \frac{1}{2}\left(\frac{\tau^3}{3}\right) + \frac{1.3}{2.4}\left(\frac{\tau^5}{5}\right) + \cdots \dots , \tau^2 < 1\right]$$

$$tan^{-1}\tau = \left[\tau - \frac{\tau^3}{3} + \frac{\tau^5}{5} - \cdots \dots \dots \dots \dots \dots \dots \dots ..\right]$$

$$log(1+\tau) = \left[\tau - \frac{\tau^2}{2} + \frac{\tau^3}{3} - \frac{\tau^4}{4} + \cdots , -1 < \tau \le 1\right]$$

$$log(1-\tau) = \left[-\tau - \frac{\tau^2}{2} - \frac{\tau^3}{3} - \frac{\tau^4}{4} - \cdots , -1 \le \tau < 1\right]$$

$$\frac{1}{(1-\tau)} = [1 + \tau + \tau^2 + \tau^3 + \cdots \dots \dots , |\tau| < 1]$$

$$\frac{1}{(1+\tau)} = [1 - \tau + \tau^2 - \tau^3 + \cdots \dots \dots \dots , |\tau| < 1]$$

$$\frac{1}{(1-\tau)^2} = [1 + 2\tau + 3\tau^2 + 4\tau^3 + \cdots \dots \dots , |\tau| < 1]$$

$$\frac{1}{(1-\tau)^3} = [1 + 3\tau + 6\tau^2 + 10\tau^3 + \cdots \ldots, |\tau| < 1]$$

$$(1+\tau)^{\frac{1}{2}} = \left[1 + \frac{\tau}{2} - \frac{\tau^2}{8} + \frac{\tau^3}{16} - \cdots \ldots \ldots \ldots \ldots, |\tau| < 1\right]$$

$$(1+\tau)^{-\frac{1}{2}} = \left[1 - \frac{\tau}{2} + \frac{3\tau^2}{8} - \frac{5\tau^3}{16} + \cdots \ldots \ldots \ldots, |\tau| < 1\right]$$

III. METHOD OF TAYLOR'S SERIES FOR THE SOLUTION OF NON-LINEAR FIRST KIND V.I.E.

The non-linear first kind V.I.E. is given by [19, 21]

$$f(\tau) = \delta \int_0^\tau K(\tau,t)\big(\varphi(t)\big)^m dt \qquad (1)$$

where

$$\left.\begin{array}{c} \varphi(t) = unknown\ function \\ f(\tau) = known\ function\ (perturbation\ function) \\ \delta = non-zero\ parameter \\ K(\tau,t) = \ kernel\ of\ integral\ equation \\ m = positive\ integer > 1 \end{array}\right\}$$

Suppose the solution $\varphi(\tau)$ of equation (1) is analytic so it can be represent in the form of Taylor's series as

$$\varphi(\tau) = \sum_{n=0}^{\infty} \beta_n \tau^n \qquad (2)$$

Use equation (2) in equation (1), we have

$$T(f(\tau)) = \delta \int_0^\tau K(\tau,t)\left(\sum_{n=0}^{\infty} \beta_n \tau^n\right)^m dt \qquad (3)$$

where $T(f(\tau))$ is the Taylor series expansion of the function $f(\tau)$.

Equation (3) can be written as

$$\left[T(f(\tau)) = \delta \int_0^\tau K(\tau,t)\begin{pmatrix} \beta_0 + \beta_1 t \\ +\beta_2 t^2 + \beta_3 t^3 \\ +\beta_4 t^4 + \beta_5 t^5 \\ + \cdots \ldots \ldots .. \end{pmatrix}^m dt\right] \qquad (4)$$

On simplification, (4) gives a system of algebraic equations in terms of $(\beta_0, \beta_1, \beta_2, \beta_3, \ldots \ldots \ldots \ldots)$. After solving this system, we get a chain of coefficients namely $(\beta_0, \beta_1, \beta_2, \beta_3, \ldots \ldots \ldots \ldots)$.The required solution of equation (1) may be obtained by using these coefficients in equation (2).

Example: 3.1 Consider the following non-linear first kind V.I.E.

$$\frac{1}{4}e^{2\tau} - \frac{1}{2}\tau - \frac{1}{4} = \int_0^\tau (\tau - t)\,(\varphi(t))^2 dt \qquad (5)$$

Suppose the solution $\varphi(\tau)$ of equation (5) is analytic so it can be represent in the form of Taylor's series as

$$\varphi(\tau) = \sum_{n=0}^{\infty} \beta_n \tau^n \qquad (6)$$

Use equation (6) in equation (5), we have

$$\left[\begin{array}{c} \frac{1}{4}\left\{\begin{array}{c} 1 + \frac{2\tau}{1!} + \frac{4\tau^2}{2!} + \frac{8\tau^3}{3!} \\ + \frac{16\tau^4}{4!} + \frac{32\tau^5}{5!} \\ + \frac{64\tau^6}{6!} + \cdots \ldots \ldots . \end{array}\right\} - \frac{1}{2}\tau - \frac{1}{4} \\ = \int_0^\tau (\tau - t)\left(\sum_{n=0}^{\infty} \beta_n \tau^n\right)^2 dt \end{array}\right] \qquad (7)$$

Equation (7) can be written as

$$\left[\begin{array}{c} \frac{\tau^2}{2} + \frac{\tau^3}{3} \\ + \frac{\tau^4}{6} + \frac{\tau^5}{15} \\ + \frac{\tau^6}{45} + \cdots \ldots .. \end{array}\right] = \left[\int_0^\tau (\tau - t)\begin{pmatrix} \beta_0 + \beta_1 t \\ +\beta_2 t^2 + \beta_3 t^3 \\ +\beta_4 t^4 + \beta_5 t^5 \\ + \cdots \ldots \ldots .. \end{pmatrix}^2 dt\right]$$

$$\Rightarrow \left[\begin{array}{c} \frac{\tau^2}{2} + \frac{\tau^3}{3} \\ + \frac{\tau^4}{6} + \frac{\tau^5}{15} \\ + \frac{\tau^6}{45} + \cdots \ldots .. \end{array}\right]$$

$$= \left[\int_0^\tau (\tau - t)\left\{\begin{array}{c} {\beta_0}^2 + 2\beta_0\beta_1 t \\ +\left({\beta_1}^2 + 2\beta_0\beta_2\right) t^2 \\ +(2\beta_1\beta_2 + 2\beta_0\beta_3)t^3 \\ +\begin{pmatrix} {\beta_2}^2 \\ +2\beta_1\beta_3 + 2\beta_0\beta_4 \end{pmatrix} t^4 \\ +(2\beta_2\beta_3 + 2\beta_1\beta_4 + 2\beta_0\beta_5)t^5 \\ + \cdots \ldots \end{array}\right\}\right] dt$$

$$\Rightarrow \left[\frac{\tau^2}{2} + \frac{\tau^3}{3} + \frac{\tau^4}{6} + \frac{\tau^5}{15} + \frac{\tau^6}{45} + \cdots \ldots ..\right]$$

$$= \tau\left[\begin{array}{c} {\beta_0}^2\tau + \beta_0\beta_1\,\tau^2 \\ +\left({\beta_1}^2 + 2\beta_0\beta_2\right)\frac{\tau^3}{3} \\ +(2\beta_1\beta_2 + 2\beta_0\beta_3)\frac{\tau^4}{4} \\ +\left({\beta_2}^2 + 2\beta_1\beta_3 + 2\beta_0\beta_4\right)\frac{\tau^5}{5} \\ +(2\beta_2\beta_3 + 2\beta_1\beta_4 + 2\beta_0\beta_5)\frac{\tau^6}{6} + \cdots \ldots \ldots \end{array}\right]$$

$$- \left[\begin{array}{c} {\beta_0}^2\frac{\tau^2}{2} + 2\beta_0\beta_1\frac{\tau^3}{3} \\ +\left({\beta_1}^2 + 2\beta_0\beta_2\right)\frac{\tau^4}{4} \\ +(2\beta_1\beta_2 + 2\beta_0\beta_3)\frac{\tau^5}{5} \\ +\left({\beta_2}^2 + 2\beta_1\beta_3 + 2\beta_0\beta_4\right)\frac{\tau^6}{6} \\ + \cdots \ldots \ldots \end{array}\right] \qquad (8)$$

Now on simplification, (8) gives a system of following algebraic equations

$$\left.\begin{aligned} \frac{1}{2} &= {\beta_0}^2 - \frac{{\beta_0}^2}{2} \\ \frac{1}{3} &= \beta_0\beta_1 - \frac{2\beta_0\beta_1}{3} \\ \frac{1}{6} &= \frac{({\beta_1}^2+2\beta_0\beta_2)}{3} - \frac{({\beta_1}^2+2\beta_0\beta_2)}{4} \\ \frac{1}{15} &= \frac{(2\beta_1\beta_2+2\beta_0\beta_3)}{4} - \frac{(2\beta_1\beta_2+2\beta_0\beta_3)}{5} \\ \frac{1}{45} &= \frac{({\beta_2}^2+2\beta_1\beta_3+2\beta_0\beta_4)}{5} - \frac{({\beta_2}^2+2\beta_1\beta_3+2\beta_0\beta_4)}{6} \end{aligned}\right\} \quad (9)$$

After solving the system (9), we get

$$\left.\begin{aligned} \beta_0 &= 1 \\ \beta_1 &= 1 \\ \beta_2 &= \frac{1}{2} \\ \beta_3 &= \frac{1}{6} \\ \beta_4 &= \frac{1}{24} \end{aligned}\right\} \text{ and } \begin{aligned} \beta_0 &= -1 \\ \beta_1 &= -1 \\ \beta_2 &= -\frac{1}{2} \\ \beta_3 &= -\frac{1}{6} \\ \beta_4 &= -\frac{1}{24} \end{aligned} \quad (10)$$

Using equation (10) in equation (6), we get the required solutions of equation (5) given by

$$\varphi(\tau) = \begin{bmatrix} 1 + 1.\tau + \left(\frac{1}{2}\right)\tau^2 \\ + \left(\frac{1}{6}\right)\tau^3 + \left(\frac{1}{24}\right)\tau^4 + \cdots \ldots \ldots \end{bmatrix} = e^{\tau}$$

and

$$\varphi(\tau) = \begin{bmatrix} -1 + (-1).\tau \\ + \left(-\frac{1}{2}\right)\tau^2 + \left(-\frac{1}{6}\right)\tau^3 \\ + \left(-\frac{1}{24}\right)\tau^4 + \cdots \ldots \ldots \end{bmatrix} = -e^{\tau}.$$

Example: 3.2 Consider the following non-linear first kind V.I.E.

$$\frac{\tau^2}{2} + \frac{\tau^3}{2} + \frac{\tau^4}{4} + \frac{\tau^5}{20} = \int_0^{\tau} (\tau - t)(\varphi(t))^3 dt \quad (11)$$

Suppose the solution $\varphi(\tau)$ of equation (11) is analytic so it can be represent in the form of Taylor's series as

$$\varphi(\tau) = \sum_{n=0}^{\infty} \beta_n \tau^n \quad (12)$$

Use equation (12) in equation (11), we have

$$\frac{\tau^2}{2} + \frac{\tau^3}{2} + \frac{\tau^4}{4} + \frac{\tau^5}{20} = \int_0^{\tau} (\tau - t)\left(\sum_{n=0}^{\infty} \beta_n t^n\right)^3 dt \quad (13)$$

Equation (13) can be written as

$$\begin{bmatrix} \frac{\tau^2}{2} + \frac{\tau^3}{2} \\ + \frac{\tau^4}{4} + \frac{\tau^5}{20} \\ = \int_0^{\tau} (\tau - t) \begin{pmatrix} \beta_0 + \beta_1 t \\ + \beta_2 t^2 + \beta_3 t^3 \\ + \beta_4 t^4 + \beta_5 t^5 + \cdots \ldots \ldots \end{pmatrix}^3 dt \end{bmatrix}$$

$$\Rightarrow \frac{\tau^2}{2} + \frac{\tau^3}{2} + \frac{\tau^4}{4} + \frac{\tau^5}{20} =$$

$$\int_0^{\tau} (\tau - t) \begin{bmatrix} {\beta_0}^3 + 3{\beta_0}^2\beta_1 t \\ + \left(3{\beta_0}^2\beta_2 + 3{\beta_1}^2\beta_0\right) t^2 \\ + \begin{pmatrix} {\beta_1}^3 + 3{\beta_0}^2\beta_3 \\ + 6\beta_0\beta_1\beta_2 \end{pmatrix} t^3 \\ + \begin{pmatrix} 3{\beta_0}^2\beta_4 + 3{\beta_1}^2\beta_2 \\ + 3{\beta_2}^2\beta_0 + 6\beta_0\beta_1\beta_3 \end{pmatrix} t^4 \\ + \begin{pmatrix} 3{\beta_0}^2\beta_5 + 3{\beta_1}^2\beta_3 \\ + 3{\beta_2}^2\beta_1 + 6\beta_0\beta_1\beta_4 \\ + 6\beta_0\beta_2\beta_3 \end{pmatrix} t^5 + \cdots \ldots \end{bmatrix} dt$$

$$\Rightarrow \frac{\tau^2}{2} + \frac{\tau^3}{2} + \frac{\tau^4}{4} + \frac{\tau^5}{20}$$

$$= \tau \begin{bmatrix} {\beta_0}^3\tau + \frac{3}{2}{\beta_0}^2\beta_1\,\tau^2 \\ + \frac{1}{3}\left(3{\beta_0}^2\beta_2 + 3{\beta_1}^2\beta_0\right)\tau^3 \\ + \frac{1}{4}\left({\beta_1}^3 + 3{\beta_0}^2\beta_3 + 6\beta_0\beta_1\beta_2\right)\tau^4 \\ + \begin{pmatrix} 3{\beta_0}^2\beta_4 + 3{\beta_1}^2\beta_2 \\ + 3{\beta_2}^2\beta_0 + 6\beta_0\beta_1\beta_3 \end{pmatrix} \frac{\tau^5}{5} \\ + \begin{pmatrix} 3{\beta_0}^2\beta_5 + 3{\beta_1}^2\beta_3 \\ + 3{\beta_2}^2\beta_1 + 6\beta_0\beta_1\beta_4 + 6\beta_0\beta_2\beta_3 \end{pmatrix} \frac{\tau^6}{6} + \cdots \ldots \end{bmatrix}$$

$$- \begin{bmatrix} {\beta_0}^3\frac{\tau^2}{2} + {\beta_0}^2\beta_1\,\tau^3 \\ + \left(3{\beta_0}^2\beta_2 + 3{\beta_1}^2\beta_0\right)\frac{\tau^4}{4} \\ + \left({\beta_1}^3 + 3{\beta_0}^2\beta_3 + 6\beta_0\beta_1\beta_2\right)\frac{\tau^5}{5} \\ + \begin{pmatrix} 3{\beta_0}^2\beta_4 + 3{\beta_1}^2\beta_2 \\ + 3{\beta_2}^2\beta_0 + 6\beta_0\beta_1\beta_3 \end{pmatrix} \frac{\tau^6}{6} \\ + \begin{pmatrix} 3{\beta_0}^2\beta_5 + 3{\beta_1}^2\beta_3 \\ + 3{\beta_2}^2\beta_1 + 6\beta_0\beta_1\beta_4 + 6\beta_0\beta_2\beta_3 \end{pmatrix} \frac{\tau^7}{7} + \cdots \ldots \end{bmatrix} \quad (14)$$

Now on simplification, (14) gives a system of following algebraic equations

$$\left.\begin{array}{r}\frac{1}{2} = {\beta_0}^3 - \frac{{\beta_0}^3}{2}\\ \frac{1}{2} = \frac{3}{2}{\beta_0}^2\beta_1 - {\beta_0}^2\beta_1\\ \frac{1}{4} = \begin{bmatrix}\frac{(3{\beta_0}^2\beta_2+3{\beta_1}^2\beta_0)}{3}\\ -\frac{(3{\beta_0}^2\beta_2+3{\beta_1}^2\beta_0)}{4}\end{bmatrix}\\ \frac{1}{20} = \begin{bmatrix}\frac{({\beta_1}^3+3{\beta_0}^2\beta_3+6\beta_0\beta_1\beta_2)}{4}\\ -\frac{({\beta_1}^3+3{\beta_0}^2\beta_3+6\beta_0\beta_1\beta_2)}{5}\end{bmatrix}\\ 0 = \begin{bmatrix}\frac{(3{\beta_0}^2\beta_4+3{\beta_1}^2\beta_2+3{\beta_2}^2\beta_0+6\beta_0\beta_1\beta_3)}{5}\\ -\frac{(3{\beta_0}^2\beta_4+3{\beta_1}^2\beta_2+3{\beta_2}^2\beta_0+6\beta_0\beta_1\beta_3)}{6}\end{bmatrix}\\ 0 = \begin{bmatrix}\frac{(3{\beta_0}^2\beta_5+3{\beta_1}^2\beta_3+3{\beta_2}^2\beta_1+6\beta_0\beta_1\beta_4+6\beta_0\beta_2\beta_3)}{6}\\ -\frac{(3{\beta_0}^2\beta_5+3{\beta_1}^2\beta_3+3{\beta_2}^2\beta_1+6\beta_0\beta_1\beta_4+6\beta_0\beta_2\beta_3)}{7}\end{bmatrix}\end{array}\right\} \quad (15)$$

After solving the system (15), we get

$$\left.\begin{array}{l}\beta_0 = 1\\ \beta_1 = 1\\ \beta_2 = 0\\ \beta_3 = 0\\ \beta_4 = 0\\ \beta_5 = 0\end{array}\right\} \quad (16)$$

Using equation (16) in equation (12), we get the required solutions of equation (11) given by

$$\varphi(\tau) = \begin{bmatrix}1 + 1.\tau + 0.\tau^2\\ +0.\tau^3 + 0.\tau^4 + 0.\tau^5 + \cdots \ldots \ldots .. \end{bmatrix} = 1 + \tau.$$

IV. CONCLUSIONS

In the present paper, authors fruitfully discussed the Taylor series method for determining the solution of non-linear first kind V.I.E. The complete methodology explained by taking numerical examples. Results of numerical examples depict that Taylor series method is very effective method for determining the solution of non-linear first kind V.I.E. without large computational work.

REFERENCES

[1]. Chauhan, R. and Aggarwal, S., Laplace transform for convolution type linear Volterra integral equation of second kind, Journal of Advanced Research in Applied Mathematics and Statistics, 4(3&4), 1-7, 2019.

[2]. Aggarwal, S., Chauhan, R. and Sharma, N., A new application of Kamal transform for solving linear Volterra integral equations, International Journal of Latest Technology in Engineering, Management & Applied Science, 7(4), 138-140, 2018.

[3]. Aggarwal, S., Chauhan, R. and Sharma, N., A new application of Mahgoub transform for solving linear Volterra integral equations, Asian Resonance, 7(2), 46-48, 2018.

[4]. Aggarwal, S., Sharma, N. and Chauhan, R., Solution of linear Volterra integral equations of second kind using Mohand transform, International Journal of Research in Advent Technology, 6(11), 3098-3102, 2018.

[5]. Aggarwal, S., Sharma, N. and Chauhan, R., A new application of Aboodh transform for solving linear Volterra integral equations, Asian Resonance, 7(3), 156-158, 2018.

[6]. Aggarwal, S. and Sharma, N., Laplace transform for the solution of first kind linear Volterra integral equation, Journal of Advanced Research in Applied Mathematics and Statistics, 4(3&4), 16-23, 2019.

[7]. Aggarwal, S., Sharma, N. and Chauhan, R., Application of Kamal transform for solving linear Volterra integral equations of first kind, International Journal of Research in Advent Technology, 6(8), 2081-2088, 2018.

[8]. Aggarwal, S., Sharma, N. and Chauhan, R., Application of Mahgoub transform for solving linear Volterra integral equations of first kind, Global Journal of Engineering Science and Researches, 5(9), 154-161, 2018.

[9]. Aggarwal, S., Sharma, N. and Chauhan, R., Application of Aboodh transform for solving linear Volterra integral equations of first kind, International Journal of Research in Advent Technology, 6(12), 3745-3753, 2018.

[10]. Aggarwal, S., Chauhan, R. and Sharma, N., Application of Elzaki transform for solving linear Volterra integral equations of first kind, International Journal of Research in Advent Technology, 6(12), 3687-3692, 2018.

[11]. Aggarwal, S., Gupta, A.R. and Sharma, S.D., A new application of Shehu transform for handling Volterra integral equations of first kind, International Journal of Research in Advent Technology, 7(4), 439-445, 2019.

[12]. Sharma, N. and Aggarwal, S., Laplace transform for the solution of Abel's integral equation, Journal of Advanced Research in Applied Mathematics and Statistics, 4(3&4), 8-15, 2019.

[13]. Aggarwal, S. and Sharma, S.D., Application of Kamal transform for solving Abel's integral equation, Global Journal of Engineering Science and Researches, 6(3), 82-90, 2019.

[14]. Aggarwal, S., Sharma, S.D. and Gupta, A.R., A new application of Mohand transform for handling Abel's integral equation, Journal of Emerging Technologies and Innovative Research, 6(3), 600-608, 2019.

[15]. Aggarwal, S. and Sharma, S.D., Solution of Abel's integral equation by Aboodh transform method, Journal of Emerging Technologies and Innovative Research, 6(4), 317-325, 2019.

[16]. Aggarwal, S. and Gupta, A.R., Sumudu transform for the solution of Abel's integral equation, Journal of Emerging Technologies and Innovative Research, 6(4), 423-431, 2019.

[17]. Aggarwal, S. and Gupta, A.R., Shehu transform for solving Abel's integral equation, Journal of Emerging Technologies and Innovative Research, 6(5), 101-110, 2019.

[18]. Aggarwal, S. and Bhatnagar, K., Solution of Abel's integral equation using Sadik transform, Asian Resonance, Vol. 8 No. 2, (Part-1), 57-63, April 2019.

[19]. Wazwaz, A.M., A first course in integral equations, World Scientific, Singapore, 1997.

[20]. Moiseiwitsch, B.L., Integral equations, Longman, London and New York, 1977.

[21]. Wazwaz, A.M., Linear and nonlinear integral equations: Methods and applications, Springer, 2011.

AUTHORS PROFILE

Dr. Sudhanshu Aggarwal has received his Ph.D. degree from C.C.S. University, Meerut in 2019. He has also qualified CSIR NET examination (June-2010, June-2012, June-2013, June-2014 and June-2015) in Mathematical Sciences. He is working as an Assistant Professor in National P.G. College Barhalganj, Gorakhpur. He is equipped with an extraordinary caliber and appreciable academic potency. His fields of interest include Integral Transform Methods, Differential and Partial Differential Equations, Integral Equations, Vibration of Plates, Theory of Elasticity and Number Theory. He has published many research papers in national and international journals.

Swarg Deep Sharma is working as Assistant Professor in the Department of Mathematics at Nand Lal Singh College Jaitpur Daudpur Constituent of Jai Prakash University Chhapra, Bihar. He has also qualified CSIR NET examination (June-2007) in Mathematical Sciences. He has published many research papers in National and International Journals. His fields of interest include Integral Transform Methods and Integral Equations.

Dr. Anjana Rani Gupta is working as Professor and Head in Department of Mathematics at Noida Institute of Engineering & Technology Greater Noida. She has completed her Ph.D. degree from IIT Roorkee. She has more than 20 years of teaching experience at IIT Roorkee and various Engineering Colleges affiliated to UPTU. She has published many research papers in reputed journals and also a member of different Mathematical Societies.