Sawi Transform of Bessel's Functions with Application for Evaluating Definite Integrals

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Abstract: **Bessel's functions have many applications to solve the problems of mathematical physics, nuclear physics, acoustics, radio physics, atomic physics, engineering and sciences such as flux distribution in a nuclear reactor, vibrations, fluid mechanics, hydrodynamics, stress analysis, heat transfer etc. In this paper, authors present Sawi transform of Bessel's functions with application for evaluating definite integrals. Results show that Sawi transform provides the value of definite integrals, which contains Bessel's functions in the integrand, in a very short time and without large calculation work.**

Keywords: **Bessel function, Sawi transform, Convolution theorem, Inverse Sawi transform, Definite integral.**

I. INTRODUCTION

Nowadays, integral transformations are one of the mostly used mathematical techniques to determine the answers of advance problems of space, science, technology and engineering. The most important feature of these transformations is providing the exact (analytical) solution of the problem without large calculation work. Aggarwal and other scholars [1-8] used different integral transformations (Mahgoub, Aboodh, Shehu, Elzaki, Mohand, Kamal) and determined the analytical solutions of first and second kind Volterra integral equations. Solutions of the problems of Volterra integro-differential equations of second kind are given by Aggarwal et al. [9-11] with the help of Mahgoub, Kamal and Aboodh transformations. In the year 2018, Aggarwal with other scholars [12-13] determined the solutions of linear partial integro-differential equations using Mahgoub and Kamal transformations. Aggarwal et al. [14-20] used Sawi; Mohand; Kamal; Shehu; Elzaki; Laplace and Mahgoub transformations and determined the solutions of advance problems of population growth and decay by the help of their mathematical models. Aggarwal et al. [21-26] defined dualities relations of many advance integral transformations. Comparative studies of Mohand and other integral transformations are given by Aggarwal et al. [27-31]. Aggarwal et al. [32-39] defined Elzaki; Aboodh; Shehu; Sumudu; Mohand; Kamal; Mahgoub and Laplace transformations of error function with applications. The solutions of ordinary differential equations with variable coefficients are given by Aggarwal et al. [40] using Mahgoub transform. Aggarwal et al. [41-45] used different integral transformations and determined the solutions of Abel's integral equations. Aggarwal et al. [46-49] worked on Bessel's functions and determined their Mohand; Aboodh; Mahgoub and Elzaki transformations. Chaudhary et al. [50] gave the connections between Aboodh transform and some useful integral transforms. Aggarwal et al. [51-52] used Elzaki and Kamal transforms for solving linear Volterra integral equations of first kind. Solution of population growth and decay problems was given by Aggarwal et al. [53-54] by using Aboodh and Sadik transformations respectively. Aggarwal and Sharma [55] defined Sadik transform of error function. Application of Sadik transform for handling linear Volterra integro-differential equations of second kind was given by Aggarwal et al. [56]. Aggarwal and Bhatnagar [57] gave the solution of Abel's integral equation using Sadik transform. A comparative study of Mohand and Mahgoub transforms was given by Aggarwal [58]. Aggarwal [59] defined Kamal transform of Bessel's functions. Chauhan and Aggarwal [60] used Laplace transform and solved convolution type linear Volterra integral equation of second kind. Sharma and Aggarwal [61] applied Laplace transform and determined the solution of Abel's integral equation. Laplace transform for the solution of first kind linear Volterra integral equation was given by Aggarwal and Sharma [62]. Mishra et al. [63] defined the relationship between Sumudu and some efficient integral transforms.

In the advance time, Bessel's functions appear in solving many problems of sciences and engineering together with many equations such as heat equation, wave equation, Laplace equation, Schrodinger equation, Helmholtz equation in cylindrical or spherical coordinates.

The object of the present paper is to determine Sawi transform of Bessel's functions and determine the value of definite integrals which contains Bessel's functions in the integrand.

II. DEFINITION OF SAWI TRANSFORM

Sawi transform of the function $F(t), t \geq 0$ was proposed by Mahgoub [64], in 2019 as:

$\left[S\{F(t)\} = \frac{1}{v^2}\int_0^\infty F(t)e^{-\frac{t}{v}}dt = R(v), \; 0 < k_1 \leq v \leq k_2 \right]$, where operator S is called the Sawi transform operator.

The Sawi transform of the function $F(t)$ for $t \geq 0$ exist if $F(t)$ is piecewise continuous and of exponential order. The mention two conditions are the only sufficient conditions for the existence of Sawi transforms of the function $F(t)$. He developed this transform for solving linear ordinary differential equations with constant coefficients.

Linearity property of Sawi transforms [14]

If $S\{F(t)\} = H(v)$ and $S\{G(t)\} = I(v)$ then

$S\{aF(t) + bG(t)\} = aS\{F(t)\} + bS\{G(t)\}$

$\Rightarrow S\{aF(t) + bG(t)\} = aH(v) + bI(v)$, where a, b are arbitrary constants.

Proof: By the definition of Sawi transform, we have

$$S\{F(t)\} = \frac{1}{v^2}\int_0^\infty F(t)e^{-\frac{t}{v}}dt$$

$$\Rightarrow \left[S\{aF(t) + bG(t)\} = \frac{1}{v^2}\int_0^\infty [aF(t) + bG(t)]e^{-\frac{t}{v}}dt \right]$$

$$\Rightarrow \left[S\{aF(t) + bG(t)\} = a\left[\frac{1}{v^2}\int_0^\infty F(t)e^{-\frac{t}{v}}dt\right] + b\left[\frac{1}{v^2}\int_0^\infty G(t)e^{-\frac{t}{v}}dt\right] \right]$$

$$\Rightarrow S\{aF(t) + bG(t)\} = aS\{F(t)\} + bS\{G(t)\}$$

$\Rightarrow S\{aF(t) + bG(t)\} = aH(v) + bI(v)$, where a, b are arbitrary constants.

Change of scale property of Sawi transforms

If $S\{F(t)\} = R(v)$ then $S\{F(at)\} = aR(av)$.

Proof: By the definition of Sawi transform, we have

$$S\{F(t)\} = \frac{1}{v^2}\int_0^\infty F(t)e^{-\frac{t}{v}}dt$$

$$\Rightarrow S\{F(at)\} = \frac{1}{v^2}\int_0^\infty F(at)e^{-\frac{t}{v}}dt$$

Put $at = p \Rightarrow adt = dp$ in above equation, we have

$$S\{F(at)\} = \frac{1}{av^2}\int_0^\infty F(p)e^{\frac{-p}{av}}dp$$

$$\Rightarrow S\{F(at)\} = a\left[\frac{1}{(av)^2}\int_0^\infty F(p)e^{\frac{-p}{av}}dp\right]$$

$$\Rightarrow S\{F(at)\} = aR(av).$$

Sawi transform of the derivatives of the function $F(t)$ [14, 64]

If $S\{F(t)\} = R(v)$ then

a) $S\{F'(t)\} = \frac{R(v)}{v} - \frac{F(0)}{v^2}$

b) $S\{F''(t)\} = \frac{R(v)}{v^2} - \frac{F(0)}{v^3} - \frac{F'(0)}{v^2}$

c) $S\{F^{(n)}(t)\} = \frac{R(v)}{v^n} - \frac{F(0)}{v^{n+1}} - \frac{F'(0)}{v^n} - \cdots \ldots - \frac{F^{(n-1)}(0)}{v^2}$.

Convolution theorem for Sawi transforms

If $S\{F(t)\} = H(v)$ and $S\{G(t)\} = I(v)$ then

$S\{F(t) * G(t)\} = v^2 S\{F(t)\}S\{G(t)\} = v^2 H(v)I(v)$

Proof: By the definition of Sawi transform, we have

$$S\{F(t)\} = \frac{1}{v^2}\int_0^\infty F(t)e^{-\frac{t}{v}}dt$$

$$\Rightarrow S\{F(t) * G(t)\} = \frac{1}{v^2}\int_0^\infty [F(t) * G(t)]e^{-\frac{t}{v}}dt$$

Now, by the definition of convolution of two functions, we have

$$\Rightarrow \left[S\{F(t) * G(t)\} = \frac{1}{v^2}\int_0^\infty \left[\int_0^t F(x)G(t-x)dx\right]e^{-\frac{t}{v}}dt \right]$$

By changing the order of integration, we have

$$\Rightarrow \left[S\{F(t) * G(t)\} = \frac{1}{v^2}\int_0^\infty F(x)\left[\int_x^\infty G(t-x)e^{-\frac{t}{v}}dt\right]dx \right]$$

Put $t - x = u$ so that $dt = du$ in above equation, we have

$$\Rightarrow \left[S\{F(t) * G(t)\} = \frac{1}{v^2}\int_0^\infty F(x)\left[\int_0^\infty G(u)e^{-\frac{(x+u)}{v}}du\right]dx \right]$$

$$\Rightarrow \left[S\{F(t) * G(t)\} = \frac{1}{v^2}\int_0^\infty F(x)e^{-\frac{x}{v}}\left[\int_0^\infty G(u)e^{-\frac{u}{v}}du\right]dx \right]$$

$$\Rightarrow S\{F(t) * G(t)\} = \int_0^\infty F(x)e^{-\frac{x}{v}}[S\{G(t)\}]dx$$

$$\Rightarrow S\{F(t) * G(t)\} = v^2 S\{G(t)\}\left[\frac{1}{v^2}\int_0^\infty F(x)e^{-\frac{x}{v}}dx\right]$$

$$\Rightarrow \left[S\{F(t) * G(t)\} = v^2 S\{F(t)\}S\{G(t)\} = v^2 H(v)I(v) \right]$$

Table 1 Sawi transform of some elementary functions [14, 64]

S.N.	$F(t)$	$S\{F(t)\} = R(v)$
1.	1	$\frac{1}{v}$
2.	t	1
3.	t^2	$2!\,v$
4.	$t^n, n \in N$	$n!\,v^{n-1}$
5.	$t^n, n > -1$	$\Gamma(n+1)v^{n-1}$
6.	e^{at}	$\frac{1}{v(1-av)}$
7.	$sinat$	$\frac{a}{1+a^2v^2}$
8.	$cosat$	$\frac{1}{v(1+a^2v^2)}$
9.	$sinhat$	$\frac{a}{1-a^2v^2}$
10.	$coshat$	$\frac{1}{v(1-a^2v^2)}$

III. INVERSE SAWI TRANSFORM

If $S\{F(t)\} = R(v)$ then $F(t)$ is called the inverse Sawi transform of $R(v)$ and mathematically it is defined as $F(t) = S^{-1}\{R(v)\}$, where the operator S^{-1} is called the inverse Sawi transform operator.

Table 2 Inverse Sawi transform of some elementary functions [14]

S.N.	$R(v)$	$F(t) = S^{-1}\{R(v)\}$
1.	$\frac{1}{v}$	1
2.	1	t
3.	v	$\frac{t^2}{2!}$
4.	$v^{n-1}, n \in N$	$\frac{t^n}{n!}$
5.	$v^{n-1}, n > -1$	$\frac{t^n}{\Gamma(n+1)}$
6.	$\frac{1}{v(1-av)}$	e^{at}
7.	$\frac{1}{1+a^2v^2}$	$\frac{sinat}{a}$
8.	$\frac{1}{v(1+a^2v^2)}$	$cosat$
9.	$\frac{1}{1-a^2v^2}$	$\frac{sinhat}{a}$
10.	$\frac{1}{v(1-a^2v^2)}$	$coshat$

Bessel's functions of different order [46-49]

Bessel's function of order n, where $n\epsilon N$ is given by

$$J_n(t) = \frac{t^n}{2^n n!}\left\{\begin{array}{c} 1 - \frac{t^2}{2.(2n+2)} \\ + \frac{t^4}{2.4.(2n+2)(2n+4)} \\ - \frac{t^6}{2.4.6.(2n+2)(2n+4)(2n+6)} \\ + \cdots .. \end{array}\right\} \quad (1)$$

In particular, when $n = 0$, we have Bessel's function of zero order and it is denoted by $J_0(t)$and it is given by the infinite power series

$$J_0(t) = \left\{1 - \frac{t^2}{2^2} + \frac{t^4}{2^2.4^2} - \frac{t^6}{2^2.4^2.6^2} + \cdots\right\} \quad (2)$$

For $n = 1$, we have Bessel's function of order one and it is denoted by $J_1(t)$and it is given by

$$J_1(t) = \left\{\begin{array}{l} \frac{t}{2} - \frac{t^3}{2^2.4} + \frac{t^5}{2^2.4^2.6} \\ - \frac{t^7}{2^2.4^2.6^2.8} + \cdots .. \end{array}\right\} \quad (3)$$

Another form of Equation (3) is given by

$$J_1(t) = \left\{\frac{t}{2} - \frac{t^3}{2^3.2!} + \frac{t^5}{2^5.2!.3!} - \frac{t^7}{2^7.3!.4!} + \cdots\right\} \quad (4)$$

For $n = 2$, we have Bessel's function of order two and it is denoted by $J_2(t)$ and it is given by

$$J_2(t) = \left\{\begin{array}{l} \frac{t^2}{2.4} - \frac{t^4}{2^2.4.6} + \frac{t^6}{2^2.4^2.6.8} \\ - \frac{t^8}{2^2.4^2.6^2.8.10} + \cdots \end{array}\right\} \quad (5)$$

Relation between $J_0(t)$ and $J_1(t)$ [46-49]

$$\frac{d}{dt}J_0(t) = -J_1(t) \quad (6)$$

Relation between $J_0(t)$ and $J_2(t)$ [46-49]

$$J_2(t) = J_0(t) + 2J_0''(t) \quad (7)$$

IV. SAWI TRANSFORM OF BESSEL'S FUNCTIONS

In this section, authors present Sawi transform of Bessel's functions.

Sawi transform of $J_0(t)$

Taking Sawi transform of equation(2), both sides, we have

$$\left[\begin{array}{c} S\{J_0(t)\} = S\{1\} - \frac{1}{2^2}S\{t^2\} \\ + \frac{1}{2^2.4^2}S\{t^4\} - \frac{1}{2^2.4^2.6^2}S\{t^6\} + \cdots .. \end{array}\right]$$

$$= \left[\begin{array}{l} \frac{1}{v} - \frac{1}{2^2}(2!\,v) + \frac{1}{2^2.4^2}(4!\,v^3) \\ - \frac{1}{2^2.4^2.6^2}(6!\,v^5) + \cdots \ldots \ldots . \end{array}\right]$$

$$= \frac{1}{v}\left[\begin{array}{l} 1 - \frac{1}{2}(v^2) + \frac{1.3}{2.4}(v^2)^2 \\ - \frac{1.3.5}{2.4.6}(v^2)^3 + \cdots \ldots \ldots \ldots . \end{array}\right]$$

$$= \frac{1}{v}(1+v^2)^{-1/2} = \frac{1}{v\sqrt{(1+v^2)}} \quad (8)$$

Sawi transform of $J_1(t)$

Taking Sawi transform of equation(6), both sides, we have

$$S\{J_1(t)\} = -S\{J_0{}'(t)\} \quad (9)$$

Now applying the property, Sawi transform of derivative of the function on equation (9), we have

$$S\{J_1(t)\} = -\left[\frac{S\{J_0(t)\}}{v} - \frac{J_0(0)}{v^2}\right] \quad (10)$$

Using equation (2) and equation (8) in equation (10), we have

$$S\{J_1(t)\} = -\left[\frac{1}{v^2\sqrt{(1+v^2)}} - \frac{1}{v^2}\right]$$

$$S\{J_1(t)\} = \frac{1}{v^2} - \frac{1}{v^2\sqrt{(1+v^2)}} \quad (11)$$

Sawi transform of $J_2(t)$

Taking Sawi transform of equation(7), both sides, we have

$$S\{J_2(t)\} = S\{J_0(t)\} + 2S\{J_0{}''(t)\} \quad (12)$$

Now applying the property, Sawi transform of derivative of the function and using equation (8) in equation(12), we have

$$\left[S\{J_2(t)\} = \frac{1}{v\sqrt{(1+v^2)}} + 2\left\{\begin{matrix}\frac{S\{J_0(t)\}}{v^2} \\ -\frac{J_0(0)}{v^3} - \frac{J_0{}'(0)}{v^2}\end{matrix}\right\}\right] \quad (13)$$

Using equation (2), equation (6) and equation (8) in equation (10), we have

$$\left[S\{J_2(t)\} = \frac{1}{v\sqrt{(1+v^2)}} + 2\left[\begin{matrix}\frac{1}{v^2}\left\{\frac{1}{v\sqrt{(1+v^2)}}\right\} \\ -\frac{1}{v^3} + \frac{J_1(0)}{v^2}\end{matrix}\right]\right] \quad (14)$$

Using equation (3) in equation (14), we have

$$S\{J_2(t)\} = \frac{1}{v\sqrt{(1+v^2)}} + \frac{2}{v^3\sqrt{(1+v^2)}} - \frac{2}{v^3}$$

$$= \frac{v^2+2-2\sqrt{(1+v^2)}}{v^3\sqrt{(1+v^2)}} \quad (15)$$

Sawi transform of $J_0(at)$

From equation (8), Sawi transform of $J_0(t)$ is given by

$$S\{J_0(t)\} = \frac{1}{v\sqrt{(1+v^2)}}$$

Now applying change of scale property of Sawi transform, we have

$$S\{J_0(at)\} = a\left[\frac{1}{av\sqrt{(1+(av)^2)}}\right] = \left[\frac{1}{v\sqrt{(1+a^2v^2)}}\right] \quad (16)$$

Sawi transform of $J_1(at)$

From equation(11), Sawi transform of $J_1(t)$ is given by

$$S\{J_1(t)\} = \frac{1}{v^2} - \frac{1}{v^2\sqrt{(1+v^2)}}$$

Now applying change of scale property of Sawi transform, we have

$$S\{J_1(at)\} = a\left[\frac{1}{(av)^2} - \frac{1}{(av)^2\sqrt{(1+(av)^2)}}\right]$$

$$= \frac{1}{av^2}\left[1 - \frac{1}{\sqrt{(1+a^2v^2)}}\right] \quad (17)$$

Sawi transform of $J_2(at)$

From equation(15), Sawi transform of $J_2(t)$ is given by

$$S\{J_2(t)\} = \frac{v^2+2-2\sqrt{(1+v^2)}}{v^3\sqrt{(1+v^2)}}$$

Now applying change of scale property of Sawi transform, we have

$$S\{J_2(at)\} = a\left[\frac{(av)^2+2-2\sqrt{(1+(av)^2)}}{(av)^3\sqrt{(1+(av)^2)}}\right]$$

$$= \frac{1}{a^2v^3}\left[\frac{a^2v^2+2-2\sqrt{(1+a^2v^2)}}{\sqrt{(1+a^2v^2)}}\right] \quad (18)$$

V. NUMERICAL APPLICATIONS

In this section, some applications are given in order to demonstrate the effectiveness of Sawi transform of Bessel's functions for evaluating the definite integral which contain Bessel's functions in the integrand.

Application: 1 Evaluate the integral

$$I(t) = \int_0^t J_0(u)J_0(t-u)du \quad (19)$$

Applying the Sawi transform to both sides of equation (19), we have

$$S\{I(t)\} = S\left\{\int_0^t J_0(u)J_0(t-u)du\right\} \quad (20)$$

Using convolution theorem of Sawi transform on equation (20), we have

$$S\{I(t)\} = v^2 S\{J_0(t)\}S\{J_0(t)\}$$

$$= v^2.\left[\frac{1}{v\sqrt{(1+v^2)}}\right].\left[\frac{1}{v\sqrt{(1+v^2)}}\right] = \frac{1}{1+v^2} \quad (21)$$

Operating inverse Sawi transform on both sides of equation (21), we have

$$I(t) = S^{-1}\left\{\frac{1}{1+v^2}\right\} = sint \quad (22)$$

which is the required exact solution of equation (19).

Application: 2 Evaluate the integral

$$I(t) = \int_0^t J_0(u)J_1(t-u)du \quad (23)$$

Applying the Sawi transform to both sides of equation (23), we have

$$S\{I(t)\} = S\left\{\int_0^t J_0(u)J_1(t-u)du\right\} \qquad (24)$$

Using convolution theorem of Sawi transform on equation (24), we have

$$S\{I(t)\} = v^2 S\{J_0(t)\}S\{J_1(t)\}$$

$$= v^2.\left[\frac{1}{v\sqrt{(1+v^2)}}\right].\left[\frac{1}{v^2} - \frac{1}{v^2\sqrt{(1+v^2)}}\right]$$

$$= \frac{1}{v\sqrt{(1+v^2)}} - \frac{1}{v(1+v^2)} \qquad (25)$$

Operating inverse Sawi transform on both sides of equation (25), we have

$$I(t) = S^{-1}\left\{\frac{1}{v\sqrt{(1+v^2)}}\right\} - S^{-1}\left\{\frac{1}{v(1+v^2)}\right\}$$

$$= J_0(t) - cost \qquad (26)$$

which is the required exact solution of equation (23).

Application: 3 Evaluate the integral

$$I(t) = \int_0^t J_1(t-u)du \qquad (27)$$

Applying the Sawi transform to both sides of equation (27), we have

$$S\{I(t)\} = S\left\{\int_0^t J_1(t-u)du\right\} \qquad (28)$$

Using convolution theorem of Sawi transform on equation (28), we have

$$S\{I(t)\} = v^2 S\{1\}S\{J_1(t)\}$$

$$= v^2.\left[\frac{1}{v}\right].\left[\frac{1}{v^2} - \frac{1}{v^2\sqrt{(1+v^2)}}\right] = \frac{1}{v} - \frac{1}{v\sqrt{(1+v^2)}} \qquad (29)$$

Operating inverse Sawi transform on both sides of equation (29), we have

$$I(t) = S^{-1}\left\{\frac{1}{v}\right\} - S^{-1}\left\{\frac{1}{v\sqrt{(1+v^2)}}\right\} = 1 - J_0(t) \quad (30)$$

which is the required exact solution of equation (28).

VI. CONCLUSIONS

In this paper, authors successfully determined the Sawi transform of Bessel's functions with application for evaluating definite integrals. Results show that Sawi transform provides the value of definite integrals, which contains Bessel's functions in the integrand, in a very short time and without large calculation work.

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