Aboodh Transform for Solving First Kind Faltung Type Volterra Integro-Differential Equation

Sudhanshu Aggarwal[1]*, Swarg Deep Sharma[2], Aakansha Vyas[3]

[1]*Assistant Professor, Department of Mathematics, National P.G. College, Barhalganj, Gorakhpur-273402, U.P., India*

[2]*Assistant Professor, Department of Mathematics, Nand Lal Singh College Jaitpur Daudpur Constituent of Jai Prakash University Chhapra-841205, Bihar, India*

[3]*Assistant Professor, Noida Institute of Engineering & Technology, Greater Noida-201306, U.P., India*

Abstract: **Volterra integro-differential equations appear in many branches of engineering, physics, biology, astronomy, radiology and having many interesting applications such as process of glass forming, diffusion process, heat and mass transfer, growth of cells and describing the motion of satellite. In this paper, authors' present Aboodh transform for solving first kind faltung type Volterra integro-differential equation. Four numerical problems have been considered and solved using Aboodh transform for explaining the methodology of present method. Results of numerical problems show that Aboodh transform is very effective integral transform for solving first kind faltung type Volterra integro-differential equation.**

Keywords: **Volterra integro-differential equation; Aboodh transform; Faltung; Inverse Aboodh transform.**

I. INTRODUCTION

Nowadays, integral transformations are one of the mostly used mathematical techniques to determine the answers of advance problems of space, science, technology and engineering. The most important feature of these transformations is providing the exact (analytical) solution of the problem without large calculation work. Aggarwal and other scholars [1-8] used different integral transformations (Mahgoub, Aboodh, Shehu, Elzaki, Mohand, Kamal) and determined the analytical solutions of first and second kind Volterra integral equations. Solutions of the problems of Volterra integro-differential equations of second kind are given by Aggarwal et al. [9-11] with the help of Mahgoub, Kamal and Aboodh transformations. In the year 2018, Aggarwal with other scholars [12-13] determined the solutions of linear partial integro-differential equations using Mahgoub and Kamal transformations. Aggarwal et al. [14-20] used Sawi; Mohand; Kamal; Shehu; Elzaki; Laplace and Mahgoub transformations and determined the solutions of advance problems of population growth and decay by the help of their mathematical models. Aggarwal et al. [21-26] defined dualities relations of many advance integral transformations. Comparative studies of Mohand and other integral transformations are given by Aggarwal et al. [27-31]. Aggarwal et al. [32-39] defined Elzaki; Aboodh; Shehu; Sumudu; Mohand; Kamal; Mahgoub and Laplace transformations of error function with applications. The solutions of ordinary differential equations with variable coefficients are given by Aggarwal et al. [40] using Mahgoub transform. Aggarwal et al. [41-45] used different integral transformations and determined the solutions of Abel's integral equations. Aggarwal et al. [46-49] worked on Bessel's functions and determined their Mohand; Aboodh; Mahgoub and Elzaki transformations.

Chaudhary et al. [50] gave the connections between Aboodh transform and some useful integral transforms. Aggarwal et al. [51-52] used Elzaki and Kamal transforms for solving linear Volterra integral equations of first kind. Solution of population growth and decay problems was given by Aggarwal et al. [53-54] by using Aboodh and Sadik transformations respectively. Aggarwal and Sharma [55] defined Sadik transform of error function. Application of Sadik transform for handling linear Volterra integro-differential equations of second kind was given by Aggarwal et al. [56]. Aggarwal and Bhatnagar [57] gave the solution of Abel's integral equation using Sadik transform. A comparative study of Mohand and Mahgoub transforms was given by Aggarwal [58]. Aggarwal [59] defined Kamal transform of Bessel's functions. Chauhan and Aggarwal [60] used Laplace transform and solved convolution type linear Volterra integral equation of second kind. Sharma and Aggarwal [61] applied Laplace transform and determined the solution of Abel's integral equation. Laplace transform for the solution of first kind linear Volterra integral equation was given by Aggarwal and Sharma [62]. Mishra et al. [63] defined relationship between Sumudu and some efficient integral transforms.

The main aim of this paper is to determine the solution of first kind faltung type Volterra integro-differential equation with the help of Aboodh transform.

II. DEFINITION OF ABOODH TRANSFORM

The Aboodh transform of the function $G(t)$ for all $t \geq 0$ is defined as [64]

$A\{G(t)\} = \frac{1}{p}\int_0^\infty G(t)e^{-pt}\,dt = g(p)$, Where A is Aboodh transform operator.

Table 1 Fundamental Properties of Aboodh Transform [30]

S.N.	Name of Property	Mathematical Form
1	Linearity	$\begin{bmatrix} A\{aG_1(t)+bG_2(t)\} \\ = aA\{G_1(t)\}+bA\{G_2(t)\}\end{bmatrix}$
2	Change of Scale	$A\{G(at)\}=\frac{1}{a^2}g\left(\frac{p}{a}\right)$
3	Shifting	$A\{e^{at}G(t)\} = \left(\frac{p-a}{p}\right)g(p-a)$
4	First Derivative	$\begin{bmatrix} A\{G^{'}(t)\} \\ = pg(p)-\frac{1}{p}G(0)\end{bmatrix}$
5	Second Derivative	$\begin{bmatrix} A\{G^{''}(t)\}=p^2g(p) \\ -G(0)-\frac{1}{p}G^{'}(0)\end{bmatrix}$
6	nth Derivative	$\begin{bmatrix} A\{G^{(n)}(t)\}=p^n g(p) \\ -p^{n-2}G(0) \\ -p^{n-3}G^{'}(0) \\ -\cdots\cdots \\ -\frac{1}{p}G^{(n-1)}(0)\end{bmatrix}$
7	Faltung	$\begin{bmatrix} A\{G_1(t)*G_2(t)\} \\ = pA\{G_1(t)\}A\{G_2(t)\}\end{bmatrix}$

Table 2 Aboodh Transform of Frequently Encountered Functions [6, 30]

S.N.	$G(t)$	$A\{G(t)\}=g(p)$
1.	1	$\frac{1}{p^2}$
2.	t	$\frac{1}{p^3}$
3.	t^2	$\frac{2!}{p^4}$
4.	$t^n, n\in N$	$\frac{n!}{p^{n+2}}$
5.	$t^n, n>-1$	$\frac{\Gamma(n+1)}{p^{n+2}}$
6.	e^{at}	$\frac{1}{p(p-a)}$
7.	$sinat$	$\frac{a}{p(p^2+a^2)}$
8.	$cosat$	$\frac{1}{p^2+a^2}$
9.	$sinhat$	$\frac{a}{p(p^2-a^2)}$
10.	$coshat$	$\frac{1}{p^2-a^2}$

III. CONNECTION BETWEEN ABOODH AND LAPLACE TRANSFORMS [50]

If Aboodh and Laplace transforms of $G(t)$ are $g(p)$ and $h(p)$ respectively then

$g(p)=\frac{1}{p}h(p)$ and $h(p)=pg(p)$,

Where $h(p)=\int_0^\infty G(t)e^{-pt}dt=L\{G(t)\}$ and L is the Laplace transform operator.

IV. INVERSE ABOODH TRANSFORM

If $A\{G(t)\}=g(p)$ then $G(t)$ is called the inverse Aboodh transform of $g(p)$ and mathematically it is defined as

$G(t)=A^{-1}\{g(p)\}$, where A^{-1} is the inverse Aboodh transform operator.

Table 3 Inverse Aboodh Transforms of Frequently Encountered Functions [6]

S.N.	$g(p)$	$G(t)=A^{-1}\{g(p)\}$
1.	$\frac{1}{p^2}$	1
2.	$\frac{1}{p^3}$	t
3.	$\frac{1}{p^4}$	$\frac{t^2}{2!}$
4.	$\frac{1}{p^{n+2}}, n\epsilon N$	$\frac{t^n}{n!}$
5.	$\frac{1}{p^{n+2}}, n>-1$	$\frac{t^n}{\Gamma(n+1)}$
6.	$\frac{1}{p(p-a)}$	e^{at}
7.	$\frac{1}{p(p^2+a^2)}$	$\frac{sinat}{a}$
8.	$\frac{1}{p^2+a^2}$	$cosat$
9.	$\frac{1}{p(p^2-a^2)}$	$\frac{sinhat}{a}$
10.	$\frac{1}{p^2-a^2}$	$coshat$

V. ABOODH TRANSFORM FOR SOLVING FIRST KIND FALTUNG TYPE VOLTERRA INTEGRO-DIFFERENTIAL EQUATION

In this part of the paper, authors gave the solution of first kind faltung type Volterra integro-differential equation using Aboodh transform.

First kind faltung type Volterra integro-differential equation is given by

$$\left.\begin{array}{l}\int_0^t K_1(t-u)\,\omega(u)du + \\ \int_0^t K_2(t-u)\,\omega^{(n)}(u)du \\ = F(t), K_2(t-u)\neq 0\end{array}\right\} \quad (1)$$

$$\text{With} \quad \left.\begin{array}{c}\omega(0)=\delta_0, \omega^{'}(0)=\delta_1, \\ \omega^{''}(0)=\delta_2, \dots\dots, \\ \omega^{(n-1)}(0)=\delta_{n-1}\end{array}\right\} \quad (2)$$

Where
$$\left.\begin{matrix}\begin{bmatrix}K_1(t-u), K_2(t-u)\\ = faltung\ type\ kernels\\ of\ integral\ equation\end{bmatrix}\\ \begin{bmatrix}\omega(t) = unknown\\ function\end{bmatrix}\\ \begin{bmatrix}\omega^{(n)}(t) = nth\ derivative\\ of unknown\ function\end{bmatrix}\\ \begin{bmatrix}F(t) = known\\ function\end{bmatrix}\\ \begin{bmatrix}\delta_0, \delta_1, \delta_2, \ldots, \delta_{n-1}\\ = real\ numbers\end{bmatrix}\end{matrix}\right\}$$

Taking Aboodh transform of both sides of (1), we have

$$\begin{bmatrix}A\left\{\int_0^t K_1(t-u)\,\omega(u)du\right\}\\ +A\left\{\int_0^t K_2(t-u)\,\omega^{(n)}(u)du\right\}\\ = A\{F(t)\}\end{bmatrix} \qquad (3)$$

Applying faltung property of Aboodh transform on (3), we have

$$\begin{bmatrix}pA\{K_1(t)\}A\{\omega(t)\}\\ +pA\{K_2(t)\}A\left\{\omega^{(n)}(t)\right\} = A\{F(t)\}\end{bmatrix} \qquad (4)$$

Applying the property "Aboodh transform of derivative of functions" on (4), we get

$$\begin{bmatrix}pA\{K_1(t)\}A\{\omega(t)\}\\ +pA\{K_2(t)\}\begin{bmatrix}p^n A\{\omega(t)\}\\ -p^{n-2}\omega(0)\\ -p^{n-3}\omega'(0)\\ -p^{n-4}\omega''(0)\\ -\cdots\ldots\\ -\frac{1}{p}\omega^{(n-1)}(0)\end{bmatrix} = A\{F(t)\}\end{bmatrix} \qquad (5)$$

Now using (2) in (5), we have

$$\begin{bmatrix}pA\{K_1(t)\}A\{\omega(t)\}\\ +pA\{K_2(t)\}\begin{bmatrix}p^n A\{\omega(t)\}\\ -p^{n-2}\delta_0\\ -p^{n-3}\delta_1\\ -p^{n-4}\delta_2\\ -\cdots\ldots\\ -\dfrac{1}{p}\delta_{n-1}\end{bmatrix} = A\{F(t)\}\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}\begin{bmatrix}pA\{K_1(t)\}\\ +p^{n+1}A\{K_2(t)\}\end{bmatrix}A\{\omega(t)\}\\ = \begin{bmatrix}A\{F(t)\}\\ +A\{K_2(t)\}\begin{pmatrix}p^{n-1}\delta_0\\ +p^{n-2}\delta_1\\ +p^{n-3}\delta_2\\ +\cdots\ldots\\ +\delta_{n-1}\end{pmatrix}\end{bmatrix}\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}A\{\omega(t)\} = \dfrac{\begin{bmatrix}A\{F(t)\}+\\ A\{K_2(t)\}\begin{pmatrix}p^{n-1}\delta_0\\ +p^{n-2}\delta_1\\ +p^{n-3}\delta_2\\ +\cdots\ldots\\ +\delta_{n-1}\end{pmatrix}\end{bmatrix}}{\begin{bmatrix}pA\{K_1(t)\}\\ +p^{n+1}A\{K_2(t)\}\end{bmatrix}},\\ \begin{bmatrix}pA\{K_1(t)\}\\ +p^{n+1}A\{K_2(t)\}\end{bmatrix} \neq 0\end{bmatrix} \qquad (6)$$

The inverse Aboodh transform of both sides of (6) gives the required solution of first kind faltung type Volterra integro-differential equation which is given by (1) with (2).

VI. NUMERICAL PROBLEMS

In this part of the paper, some numerical problems have been considered for explaining the complete methodology.

Problem: 1 Consider the following first kind faltung type Volterra integro-differential equation

$$\begin{bmatrix}\int_0^t (t-u)\,\omega(u)du\\ +\int_0^t (t-u)^2\,\omega'(u)du\\ = 3t - 3sint\end{bmatrix} \qquad (7)$$

with $\omega(0) = 0$ (8)

Taking Aboodh transform of both sides of (7), we have

$$\begin{bmatrix}A\left\{\int_0^t (t-u)\,\omega(u)du\right\}\\ +A\left\{\int_0^t (t-u)^2\,\omega'(u)du\right\}\\ = A\{3t - 3sint\}\end{bmatrix} \qquad (9)$$

Applying faltung property of Aboodh transform on (9), we have

$$\begin{bmatrix}pA\{t\}A\{\omega(t)\} + pA\{t^2\}A\{\omega'(t)\}\\ = A\{3t - 3sint\}\\ = 3A\{t\} - 3A\{sint\}\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}\frac{1}{p^2}A\{\omega(t)\} + \frac{2}{p^3}A\{\omega'(t)\}\\ = \frac{3}{p^3} - \frac{3}{p(p^2+1)}\end{bmatrix} \qquad (10)$$

Applying the property "Aboodh transform of derivative of functions" on (10), we get

$$\begin{bmatrix}\frac{1}{p^2}A\{\omega(t)\}\\ +\frac{2}{p^3}\left[pA\{\omega(t)\} - \frac{1}{p}\omega(0)\right]\\ = \frac{3}{p^3} - \frac{3}{p(p^2+1)}\end{bmatrix} \qquad (11)$$

Now using (8) in (11), we have

$$\left[\frac{3}{p^2}A\{\omega(t)\} = \frac{3}{p^3} - \frac{3}{p(p^2+1)}\right]$$

$$\Rightarrow \begin{bmatrix} A\{\omega(t)\} \\ = \frac{1}{p} - \frac{p}{(p^2+1)} = \frac{1}{p(p^2+1)} \end{bmatrix} \quad (12)$$

Taking inverse Aboodh transform of both sides of (12), we get the required solution of (7) with (8) as

$$\left[\omega(t) = A^{-1}\left\{\frac{1}{p(p^2+1)}\right\} = sint\right].$$

Problem: 2 consider the following first kind faltung type Volterra integro-differential equation

$$\begin{bmatrix} \int_0^t sin(t-u)\,\omega(u)du \\ -\frac{1}{2}\int_0^t (t-u)\,\omega''(u)du \\ = \frac{t}{2} - \frac{tcost}{2} \end{bmatrix} \quad (13)$$

With $[\omega(0) = 0, \omega'(0) = 1]$ (14)

Taking Aboodh transform of both sides of (13), we have

$$\begin{bmatrix} A\left\{\int_0^t sin(t-u)\,\omega(u)du\right\} \\ -\frac{1}{2}A\left\{\int_0^t (t-u)\,\omega''(u)du\right\} \\ = A\left\{\frac{t}{2} - \frac{tcost}{2}\right\} \end{bmatrix} \quad (15)$$

Applying faltung property of Aboodh transform on (15), we have

$$\begin{bmatrix} pA\{sint\}A\{\omega(t)\} \\ -\frac{1}{2}pA\{t\}A\{\omega''(t)\} \\ = \frac{1}{2}A\{t\} - \frac{1}{2}A\{tcost\} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{1}{(p^2+1)}A\{\omega(t)\} \\ -\frac{1}{2}\left(\frac{1}{p^2}\right)A\{\omega''(t)\} \\ = \frac{1}{2}\left(\frac{1}{p^3}\right) - \frac{1}{2}\left(\frac{p^2-1}{p(p^2+1)^2}\right) \end{bmatrix} \quad (16)$$

Applying the property "Aboodh transform of derivative of functions" on (16), we get

$$\begin{bmatrix} \frac{1}{(p^2+1)}A\{\omega(t)\} \\ -\frac{1}{2}\left(\frac{1}{p^2}\right)\begin{bmatrix} p^2A\{\omega(t)\} \\ -\omega(0) \\ -\frac{1}{p}\omega'(0) \end{bmatrix} \\ = \frac{1}{2}\left(\frac{1}{p^3}\right) - \frac{1}{2}\left(\frac{p^2-1}{p(p^2+1)^2}\right) \end{bmatrix} \quad (17)$$

Now using (14) in (17), we have

$$\begin{bmatrix} \frac{1}{(p^2+1)}A\{\omega(t)\} \\ -\frac{1}{2}\left(\frac{1}{p^2}\right)\left[p^2A\{\omega(t)\} - \frac{1}{p}\right] \\ = \frac{1}{2}\left(\frac{1}{p^3}\right) - \frac{1}{2}\left(\frac{p^2-1}{p(p^2+1)^2}\right) \end{bmatrix}$$

$$\Rightarrow A\{\omega(t)\} = \frac{1}{p(p^2+1)} \quad (18)$$

Taking inverse Aboodh transform of both sides of (18), we get the required solution of (13) with (14) as

$$\left[\omega(t) = A^{-1}\left\{\frac{1}{p(p^2+1)}\right\} = sint\right].$$

Problem: 3 consider the following first kind faltung type Volterra integro-differential equation

$$\begin{bmatrix} \int_0^t cos(t-u)\,\omega(u)du \\ +\int_0^t sin(t-u)\,\omega'''(u)du \\ = 1 + sint - cost \end{bmatrix} \quad (19)$$

with $\begin{bmatrix} \omega(0) = 1, \\ \omega'(0) = 1, \omega''(0) = -1 \end{bmatrix}$ (20)

Taking Aboodh transform of both sides of (19), we have

$$\begin{bmatrix} A\left\{\int_0^t cos(t-u)\,\omega(u)du\right\} \\ +A\left\{\int_0^t sin(t-u)\,\omega'''(u)du\right\} \\ = A\{1 + sint - cost\} \end{bmatrix} \quad (21)$$

Applying faltung property of Aboodh transform on (21), we have

$$\begin{bmatrix} pA\{cost\}A\{\omega(t)\} \\ +pA\{sint\}A\{\omega'''(t)\} \\ = A\{1\} + A\{sint\} - A\{cost\} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \left(\frac{p}{(p^2+1)}\right)A\{\omega(t)\} \\ +\left(\frac{1}{(p^2+1)}\right)A\{\omega'''(t)\} \\ = \left(\frac{1}{p^2}\right) + \left(\frac{1}{p(p^2+1)}\right) - \left(\frac{1}{(p^2+1)}\right) \end{bmatrix} \quad (22)$$

Applying the property "Aboodh transform of derivative of functions" on (22), we get

$$\begin{bmatrix} \left(\frac{p}{(p^2+1)}\right)A\{\omega(t)\} \\ +\left(\frac{1}{(p^2+1)}\right)\begin{bmatrix} p^3A\{\omega(t)\} \\ -p\omega(0) \\ -\omega'(0) \\ -\frac{1}{p}\omega''(0) \end{bmatrix} \\ = \left(\frac{1}{p^2}\right) + \left(\frac{1}{p(p^2+1)}\right) - \left(\frac{1}{(p^2+1)}\right) \end{bmatrix} \quad (23)$$

Now using (20) in (23), we have

$$\begin{bmatrix} \left(\frac{p}{(p^2+1)}\right)A\{\omega(t)\} \\ +\left(\frac{1}{(p^2+1)}\right)\begin{bmatrix} p^3A\{\omega(t)\} \\ -p - 1 + \frac{1}{p} \end{bmatrix} \\ = \left(\frac{1}{p^2}\right) + \left(\frac{1}{p(p^2+1)}\right) - \left(\frac{1}{(p^2+1)}\right) \end{bmatrix}$$

$$\Rightarrow \left[L\{\omega(t)\} = \frac{1}{p^3} + \left(\frac{1}{(p^2+1)}\right)\right] \tag{24}$$

Taking inverse Aboodh transform of both sides of (24), we get the required solution of (19) with (20) as

$$\begin{bmatrix} \omega(t) = A^{-1}\left\{\frac{1}{p^3} + \frac{1}{(p^2+1)}\right\} \\ = A^{-1}\left\{\frac{1}{p^3}\right\} + A^{-1}\left\{\frac{1}{(p^2+1)}\right\} \end{bmatrix}$$

$$\Rightarrow \omega(t) = t + cost.$$

Problem: 4 consider the following first kind faltung type Volterra integro-differential equation

$$\begin{bmatrix} \int_0^t (t-u)^2\, \omega(u)du \\ -\frac{1}{12}\int_0^t (t-u)^3\, \omega'''(u)du = \frac{t^4}{12} \end{bmatrix} \tag{25}$$

with $\begin{bmatrix} \omega(0) = 0, \\ \omega'(0) = 3, \omega''(0) = 0 \end{bmatrix}$ (26)

Taking Aboodh transform of both sides of (25), we have

$$\begin{bmatrix} A\left\{\int_0^t (t-u)^2\, \omega(u)du\right\} \\ -\frac{1}{12}A\left\{\int_0^t (t-u)^3\, \omega'''(u)du\right\} \\ = \frac{1}{12}A\{t^4\} \end{bmatrix} \tag{27}$$

Applying faltung property of Aboodh transform on (27), we have

$$\begin{bmatrix} pA\{t^2\}A\{\omega(t)\} \\ -\frac{1}{12}pA\{t^3\}A\{\omega'''(t)\} = \frac{2}{p^6} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{2}{p^3}A\{\omega(t)\} \\ -\frac{1}{12}\left(\frac{6}{p^4}\right)A\{\omega'''(t)\} = \frac{2}{p^6} \end{bmatrix} \tag{28}$$

Applying the property "Aboodh transform of derivative of functions" on (28), we get

$$\begin{bmatrix} \frac{2}{p^3}A\{\omega(t)\} \\ -\frac{1}{2}\left(\frac{1}{p^4}\right)\begin{bmatrix} p^3A\{\omega(t)\} \\ -p\omega(0) \\ -\omega'(0) - \frac{1}{p}\omega''(0) \end{bmatrix} = \frac{2}{p^6} \end{bmatrix} \tag{29}$$

Now using (26) in (29), we have

$$\begin{bmatrix} \frac{2}{p^3}A\{\omega(t)\} \\ -\frac{1}{2}\left(\frac{1}{p^4}\right)[p^3A\{\omega(t)\} - 3] = \frac{2}{p^6} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \left[\frac{2}{p^3} - \frac{1}{2}\left(\frac{1}{p}\right)\right]A\{\omega(t)\} \\ = \frac{2}{p^6} - \frac{3}{2}\left(\frac{1}{p^4}\right) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \left[\frac{4-p^2}{2p^3}\right]A\{\omega(t)\} \\ = \frac{2}{p^6} - \frac{3}{2}\left(\frac{1}{p^4}\right) = \left[\frac{4-3p^2}{2p^6}\right] \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} A\{\omega(t)\} = \left[\frac{4-3p^2}{p^3(4-p^2)}\right] \\ = \left[\frac{1}{p^3} - \frac{2}{p(4-p^2)}\right] = \left[\frac{1}{p^3} + \frac{2}{p(p^2-4)}\right] \end{bmatrix} \tag{30}$$

Taking inverse Aboodh transform of both sides of (30), we get the required solution of (25) with (26) as

$$\begin{bmatrix} \omega(t) = A^{-1}\left\{\frac{1}{p^3} + \frac{2}{p(p^2-4)}\right\} \\ = A^{-1}\left\{\frac{1}{p^3}\right\} + 2A^{-1}\left\{\frac{1}{p(p^2-4)}\right\} \end{bmatrix}$$

$$\Rightarrow \omega(t) = t + sinh2t.$$

VII. CONCLUSIONS

In this paper, authors successfully discussed the Aboodh transform for the solution of first kind faltung type Volterra integro-differential equation and complete methodology explained by giving four numerical problems. The results of numerical problems show that the Aboodh transform is very useful integral transformation for determining the solution of first kind faltung type Volterra integro-differential equation. In future, Aboodh transform can be used for solving system of first kind faltung type Volterra integro-differential equations.

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