Application of Sawi Transform for Solving Convolution Type Volterra Integro-Differential Equation of First Kind

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Abstract: **Several situations of science, engineering, physics, biology, astronomy, radiology and statistics lead to Volterra integro-differential equations such as process of glass forming, diffusion problem, radiation transfer problem, growth of cells and describing the motion of satellite. In this paper, authors gave the application of Sawi transform for solving convolution type Volterra integro-differential equation of first kind. Some numerical problems have been considered and solved with the help of Sawi transform for explaining the complete methodology. Results of numerical problems show that Sawi transform is very effective integral transform for solving convolution type Volterra integro-differential equation of first kind.**

Keywords: **Volterra integro-differential equation; Sawi transform; Convolution; Inverse Sawi transform.**

I. INTRODUCTION

Integral transforms are very useful tool to deal with problems in applied mathematics, theoretical mechanics, statistics, mathematical physics and pharmacokinetics. The most important and attractive feature of these transforms is providing the exact (analytical) solution of the problem without large calculation work. Aggarwal and other scholars [1-8] used different integral transformations (Mahgoub, Aboodh, Shehu, Elzaki, Mohand, Kamal) and determined the analytical solutions of first and second kind Volterra integral equations. Solutions of the problems of Volterra integro-differential equations of second kind are given by Aggarwal et al. [9-11] with the help of Mahgoub, Kamal and Aboodh transformations. In the year 2018, Aggarwal with other scholars [12-13] determined the solutions of linear partial integro-differential equations using Mahgoub and Kamal transformations. Aggarwal et al. [14-20] used Sawi; Mohand; Kamal; Shehu; Elzaki; Laplace and Mahgoub transformations and determined the solutions of population growth and decay problems by the help of their mathematical models. Aggarwal et al. [21-26] defined dualities relations of many advance integral transformations. Comparative studies of Mohand and other integral transformations are given by Aggarwal et al. [27-31]. Aggarwal et al. [32-39] defined Elzaki; Aboodh; Shehu; Sumudu; Mohand; Kamal; Mahgoub and Laplace transformations of error function with applications. The solutions of ordinary differential equations with variable coefficients are given by Aggarwal et al. [40] using Mahgoub transform. Aggarwal et al. [41-45] used different integral transformations and determined the solutions of Abel's integral equations. Aggarwal et al. [46-49] worked on Bessel's functions and determined their Mohand; Aboodh; Mahgoub and Elzaki transformations. Chaudhary et al. [50] gave the connections between Aboodh transform and some useful integral transforms. Aggarwal et al. [51-52] used Elzaki and Kamal transforms for solving linear Volterra integral equations of first kind. Solution of population growth and decay problems was given by Aggarwal et al. [53-54] by using Aboodh and Sadik transformations respectively. Aggarwal and Sharma [55] defined Sadik transform of error function. Application of Sadik transform for handling linear Volterra integro-differential equations of second kind was given by Aggarwal et al. [56]. Aggarwal and Bhatnagar [57] gave the solution of Abel's integral equation using Sadik transform. A comparative study of Mohand and Mahgoub transforms was given by Aggarwal [58]. Aggarwal [59] defined Kamal transform of Bessel's functions. Chauhan and Aggarwal [60] used Laplace transform and solved convolution type linear Volterra integral equation of second kind. Sharma and Aggarwal [61] applied Laplace transform and determined the solution of Abel's integral equation. Laplace transform for the solution of first kind linear Volterra integral equation was given by Aggarwal and Sharma [62]. Mishra et al. [63] defined the relationship between Sumudu and some efficient integral transforms.

The main aim of this paper is to determine the solution of convolution type Volterra integro-differential equation of first kind with the help of Sawi transform.

II. DEFINITION OF SAWI TRANSFORM

The Sawi transform of the function $G(t)$ for all $t \geq 0$ is defined as [64]:

$S\{G(t)\} = \frac{1}{p^2}\int_0^\infty G(t)e^{-\left(\frac{t}{p}\right)}dt = g(p), k_1 \leq p \leq k_2$, where S is Sawi transform operator.

TABLE 1 FUNDAMENTAL PROPERTIES OF SAWI TRANSFORM [14]

S.N.	Name of Property	Mathematical Form
1.	Linearity	$\left[\begin{array}{l} S\{aG_1(t)+bG_2(t)\} \\ = aS\{G_1(t)\}+bS\{G_2(t)\}\end{array}\right]$
2.	Change of Scale	$S\{G(at)\} = ag(ap)$
3.	Shifting	$S\{e^{at}G(t)\} = \left(\frac{1}{1-ap}\right)^2 g\left(\frac{p}{1-ap}\right)$
4.	First Derivative	$\left[\begin{array}{l} S\{G'(t)\} \\ = \frac{1}{p}g(p) - \frac{1}{p^2}G(0)\end{array}\right]$
5.	Second Derivative	$\left[\begin{array}{l} S\{G''(t)\} = \frac{1}{p^2}g(p) \\ -\frac{1}{p^3}G(0) - \frac{1}{p^2}G'(0)\end{array}\right]$
6.	nth Derivative	$\left[\begin{array}{l} S\{G^{(n)}(t)\} \\ = \frac{1}{p^n}g(p) - \frac{1}{p^{n+1}}G(0) \\ -\frac{1}{p^n}G'(0) \ldots\ldots - \frac{1}{p^2}G^{(n-1)}(0)\end{array}\right]$
7.	Convolution	$S\{G_1(t) * G_2(t)\} = p^2 S\{G_1(t)\}S\{G_2(t)\}$

TABLE 2 SAWI TRANSFORM OF FREQUENTLY ENCOUNTERED FUNCTIONS [14, 64]

S.N.	$G(t)$	$S\{G(t)\} = g(p)$
1.	1	$\frac{1}{p}$
2.	t	1
3.	t^2	$2!\,p$
4.	$t^n, n\epsilon N$	$n!\,p^{n-1}$
5.	$t^n, n > -1$	$\Gamma(n+1)p^{n-1}$
6.	e^{at}	$\frac{1}{p(1-ap)}$
7.	$sinat$	$\frac{a}{1+a^2p^2}$
8.	$cosat$	$\frac{1}{p(1+a^2p^2)}$
9.	$sinhat$	$\frac{a}{1-a^2p^2}$
10.	$coshat$	$\frac{1}{p(1-a^2p^2)}$

Duality between Sawi and Laplace Transforms [25]

If Sawi and Laplace transformations of $G(t)$ are $g(p)$ and $h(p)$ respectively then

$g(p) = \frac{1}{p^2}h\left(\frac{1}{p}\right)$ and $h(p) = \frac{1}{p^2}g\left(\frac{1}{p}\right)$,

where $h(p) = \int_0^\infty G(t)e^{-pt}dt = L\{G(t)\}$ and L is the Laplace transform operator.

III. INVERSE SAWI TRANSFORM

If $S\{G(t)\} = g(p)$ then $G(t)$ is called the inverse Sawi transform of $g(p)$ and mathematically it is defined as

$G(t) = S^{-1}\{g(p)\}$, where S^{-1} is the inverse Sawi transform operator.

TABLE 3 INVERSE SAWI TRANSFORM OF FREQUENTLY ENCOUNTERED FUNCTIONS

S.N.	$g(p)$	$G(t) = S^{-1}\{g(p)\}$
1.	$\frac{1}{p}$	1
2.	1	t
3.	p	$\frac{t^2}{2!}$
4.	$p^{n-1}, n\epsilon N$	$\frac{t^n}{n!}$
5.	$p^{n-1}, n > -1$	$\frac{t^n}{\Gamma(n+1)}$
6.	$\frac{1}{p(1-ap)}$	e^{at}
7.	$\frac{1}{1+a^2p^2}$	$\frac{sinat}{a}$
8.	$\frac{1}{p(1+a^2p^2)}$	$cosat$
9.	$\frac{1}{1-a^2p^2}$	$\frac{sinhat}{a}$
10.	$\frac{1}{p(1-a^2p^2)}$	$coshat$

IV. APPLICATION OF SAWI TRANSFORM FOR SOLVING CONVOLUTION TYPE VOLTERRA INTEGRO-DIFFERENTIAL EQUATION OF FIRST KIND

In this part of the paper, authors determine the solution of convolution type Volterra integro-differential equation of first kind with the help of Sawi transform.

Convolution type Volterra integro-differential equation of first kind is given by

$$\left.\begin{array}{l} \int_0^t K_1(t-u)\,\omega(u)du + \\ \int_0^t K_2(t-u)\,\omega^{(n)}(u)du \\ = F(t), K_2(t-u) \neq 0 \end{array}\right\} \quad (1)$$

$$\text{with} \quad \left.\begin{array}{l} \omega(0) = \delta_0, \omega'(0) = \delta_1, \\ \omega''(0) = \delta_2, \ldots\ldots, \\ \omega^{(n-1)}(0) = \delta_{n-1} \end{array}\right\} \quad (2)$$

where $\left.\begin{matrix} \begin{bmatrix} K_1(t-u), K_2(t-u) \\ = faltung\ type\ kernels \\ of\ integral\ equation \end{bmatrix} \\ \begin{bmatrix} \omega(t) = unknown \\ function \end{bmatrix} \\ \begin{bmatrix} \omega^{(n)}(t) = nth\ derivative \\ of unknown\ function \end{bmatrix} \\ \begin{bmatrix} F(t) = known \\ function \end{bmatrix} \\ \begin{bmatrix} \delta_0, \delta_1, \delta_2, \ldots, \delta_{n-1} \\ = real\ numbers \end{bmatrix} \end{matrix}\right\}$

Taking Sawi transform of both sides of (1), we have

$$\begin{bmatrix} S\left\{\int_0^t K_1(t-u)\,\omega(u)du\right\} \\ +S\left\{\int_0^t K_2(t-u)\,\omega^{(n)}(u)du\right\} \\ = S\{F(t)\} \end{bmatrix} \quad (3)$$

Applying convolution theorem of Sawi transform on (3), we have

$$\begin{bmatrix} p^2 S\{K_1(t)\}S\{\omega(t)\} \\ +p^2 S\{K_2(t)\}S\{\omega^{(n)}(t)\} = S\{F(t)\} \end{bmatrix} \quad (4)$$

Applying the property "Sawi transform of derivative of functions" on (4), we get

$$\begin{bmatrix} p^2 S\{K_1(t)\}S\{\omega(t)\} \\ +p^2 S\{K_2(t)\} \begin{bmatrix} \frac{1}{p^n} S\{\omega(t)\} \\ -\frac{1}{p^{n+1}}\omega(0) \\ -\frac{1}{p^n}\omega'(0) \\ -\frac{1}{p^{n-1}}\omega''(0) \\ -\cdots\ldots \\ -\frac{1}{p^2}\omega^{(n-1)}(0) \end{bmatrix} = S\{F(t)\} \end{bmatrix} \quad (5)$$

Now using (2) in (5), we have

$$\begin{bmatrix} p^2 S\{K_1(t)\}S\{\omega(t)\} \\ +p^2 S\{K_2(t)\} \begin{bmatrix} \frac{1}{p^n} S\{\omega(t)\} \\ -\frac{1}{p^{n+1}}\delta_0 \\ -\frac{1}{p^n}\delta_1 \\ -\frac{1}{p^{n-1}}\delta_2 \\ -\cdots\ldots \\ -\frac{1}{p^2}\delta_{n-1} \end{bmatrix} = S\{F(t)\} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \begin{bmatrix} p^2 S\{K_1(t)\} \\ +\frac{1}{p^{n-2}} S\{K_2(t)\} \end{bmatrix} S\{\omega(t)\} \\ = \begin{bmatrix} S\{F(t)\} \\ +S\{K_2(t)\} \begin{pmatrix} \frac{1}{p^{n-1}}\delta_0 \\ +\frac{1}{p^{n-2}}\delta_1 \\ +\frac{1}{p^{n-3}}\delta_2 \\ +\cdots\ldots \\ +\delta_{n-1} \end{pmatrix} \end{bmatrix} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} S\{\omega(t)\} = \dfrac{\begin{bmatrix} S\{F(t)\} \\ +S\{K_2(t)\} \begin{pmatrix} \frac{1}{p^{n-1}}\delta_0 \\ +\frac{1}{p^{n-2}}\delta_1 \\ +\frac{1}{p^{n-3}}\delta_2 \\ +\cdots\ldots \\ +\delta_{n-1} \end{pmatrix} \end{bmatrix}}{\begin{bmatrix} p^2 S\{K_1(t)\} \\ +\frac{1}{p^{n-2}} S\{K_2(t)\} \end{bmatrix}}, \\ \begin{bmatrix} p^2 S\{K_1(t)\} \\ +\frac{1}{p^{n-2}} S\{K_2(t)\} \end{bmatrix} \neq 0 \end{bmatrix} \quad (6)$$

The inverse Sawi transform of both sides of (6) gives the required solution of given convolution type Volterra integro-differential equation of first kind.

V. NUMERICAL PROBLEMS

In this part of the paper, some numerical problems have been considered for explaining the complete methodology.

Problem: 1 Consider the following convolution type Volterra integro-differential equation of first kind

$$\begin{bmatrix} \int_0^t (t-u)\,\omega(u)du \\ +\int_0^t (t-u)^2\,\omega'(u)du \\ = 3t - 3sint \end{bmatrix} \quad (7)$$

with $\omega(0) = 0$ (8)

Taking Sawi transform of both sides of (7), we have

$$\begin{bmatrix} S\left\{\int_0^t (t-u)\,\omega(u)du\right\} \\ +S\left\{\int_0^t (t-u)^2\,\omega'(u)du\right\} \\ = S\{3t - 3sint\} \end{bmatrix} \quad (9)$$

Applying convolution theorem of Sawi transform on (9), we have

$$\begin{bmatrix} p^2 S\{t\}S\{\omega(t)\} + p^2 S\{t^2\}S\{\omega'(t)\} \\ = S\{3t - 3sint\} \\ = 3S\{t\} - 3S\{sint\} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} p^2 S\{\omega(t)\} + 2p^3 S\{\omega'(t)\} \\ = 3 - \frac{3}{1+p^2} \end{bmatrix} \quad (10)$$

Applying the property "Sawi transform of derivative of functions" on (10), we get

$$\begin{bmatrix} p^2 S\{\omega(t)\} \\ +2p^3 \left[\frac{1}{p} S\{\omega(t)\} - \frac{1}{p^2}\omega(0)\right] \\ = 3 - \frac{3}{1+p^2} \end{bmatrix} \quad (11)$$

Now using (8) in (11), we have

$$\left[3p^2 S\{\omega(t)\} = 3 - \frac{3}{1+p^2}\right]$$

$$\Rightarrow \begin{bmatrix} S\{\omega(t)\} \\ = \frac{1}{p^2} - \frac{1}{p^2(p^2+1)} = \frac{1}{(p^2+1)} \end{bmatrix} \quad (12)$$

Taking inverse Sawi transform of both sides of (12), we get the required solution of (7) with (8) as

$$\left[\omega(t) = S^{-1}\left\{\frac{1}{(p^2+1)}\right\} = sint\right].$$

Problem: 2 Consider the following convolution type Volterra integro-differential equation of first kind

$$\begin{bmatrix} \int_0^t sin(t-u)\,\omega(u)du \\ -\frac{1}{2}\int_0^t (t-u)\,\omega''(u)du \\ = \frac{t}{2} - \frac{tcost}{2} \end{bmatrix} \quad (13)$$

with $[\omega(0) = 0, \omega'(0) = 1]$ (14)

Taking Sawi transform of both sides of (13), we have

$$\begin{bmatrix} S\left\{\int_0^t sin(t-u)\,\omega(u)du\right\} \\ -\frac{1}{2} S\left\{\int_0^t (t-u)\,\omega''(u)du\right\} \\ = S\left\{\frac{t}{2} - \frac{tcost}{2}\right\} \end{bmatrix} \quad (15)$$

Applying convolution theorem of Sawi transform on (15), we have

$$\begin{bmatrix} p^2 S\{sint\} S\{\omega(t)\} \\ -\frac{1}{2} p^2 S\{t\} S\{\omega''(t)\} \\ = \frac{1}{2} S\{t\} - \frac{1}{2} S\{tcost\} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{p^2}{(p^2+1)} S\{\omega(t)\} \\ -\frac{1}{2}(p^2) S\{\omega''(t)\} \\ = \frac{1}{2} - \frac{1}{2}\left\{\frac{(1-p^2)}{(p^2+1)^2}\right\} \end{bmatrix} \quad (16)$$

Applying the property "Sawi transform of derivative of functions" on (16), we get

$$\begin{bmatrix} \frac{p^2}{(p^2+1)} S\{\omega(t)\} \\ -\frac{1}{2}(p^2) \begin{bmatrix} \frac{1}{p^2} S\{\omega(t)\} \\ -\frac{1}{p^3}\omega(0) \\ -\frac{1}{p^2}\omega'(0) \end{bmatrix} \\ = \frac{1}{2} - \frac{1}{2}\left\{\frac{(1-p^2)}{(p^2+1)^2}\right\} \end{bmatrix} \quad (17)$$

Now using (14) in (17), we have

$$\begin{bmatrix} \frac{p^2}{(p^2+1)} S\{\omega(t)\} \\ -\frac{1}{2}(p^2)\left[\frac{1}{p^2} S\{\omega(t)\} - \frac{1}{p^2}\right] \\ = \frac{1}{2} - \frac{1}{2}\left\{\frac{(1-p^2)}{(p^2+1)^2}\right\} \end{bmatrix}$$

$$\Rightarrow S\{\omega(t)\} = \frac{1}{(p^2+1)} \quad (18)$$

Taking inverse Sawi transform of both sides of (18), we get the required solution of (13) with (14) as

$$\left[\omega(t) = S^{-1}\left\{\frac{1}{(p^2+1)}\right\} = sint\right].$$

Problem: 3 Consider the following convolution type Volterra integro-differential equation of first kind

$$\begin{bmatrix} \int_0^t cos(t-u)\,\omega(u)du \\ +\int_0^t sin(t-u)\,\omega'''(u)du \\ = 1 + sint - cost \end{bmatrix} \quad (19)$$

with $\begin{bmatrix} \omega(0) = 1, \\ \omega'(0) = 1, \omega''(0) = -1 \end{bmatrix}$ (20)

Taking Sawi transform of both sides of (19), we have

$$\begin{bmatrix} S\left\{\int_0^t cos(t-u)\,\omega(u)du\right\} \\ +S\left\{\int_0^t sin(t-u)\,\omega'''(u)du\right\} \\ = S\{1 + sint - cost\} \end{bmatrix} \quad (21)$$

Applying convolution theorem of Sawi transform on (21), we have

$$\begin{bmatrix} p^2 S\{cost\} S\{\omega(t)\} \\ +p^2 S\{sint\} S\{\omega'''(t)\} \\ = S\{1\} + S\{sint\} - S\{cost\} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \left(\frac{p}{(p^2+1)}\right) S\{\omega(t)\} \\ +\left(\frac{p^2}{(p^2+1)}\right) S\{\omega'''(t)\} \\ = \frac{1}{p} + \left(\frac{1}{(p^2+1)}\right) - \left(\frac{1}{p(p^2+1)}\right) \end{bmatrix} \quad (22)$$

Applying the property "Sawi transform of derivative of functions" on (22), we get

$$\begin{bmatrix} \left(\frac{p}{(p^2+1)}\right) S\{\omega(t)\} \\ +\left(\frac{p^2}{(p^2+1)}\right) \begin{bmatrix} \frac{1}{p^3} S\{\omega(t)\} \\ -\frac{1}{p^4}\omega(0) \\ -\frac{1}{p^3}\omega'(0) \\ -\frac{1}{p^2}\omega''(0) \end{bmatrix} \\ = \frac{1}{p} + \left(\frac{1}{(p^2+1)}\right) - \left(\frac{1}{p(p^2+1)}\right) \end{bmatrix} \quad (23)$$

Now using (20) in (23), we have

$$\begin{bmatrix} \left(\frac{p}{(p^2+1)}\right) S\{\omega(t)\} \\ +\left(\frac{p^2}{(p^2+1)}\right)\left[\frac{1}{p^3} S\{\omega(t)\} - \frac{1}{p^4} - \frac{1}{p^3} + \frac{1}{p^2}\right] \\ = 1 + \left(\frac{p}{(p^2+1)}\right) - \left(\frac{1}{(p^2+1)}\right) \end{bmatrix}$$

$$\Rightarrow \left[S\{\omega(t)\} = 1 + \left(\frac{1}{p(p^2+1)}\right)\right] \quad (24)$$

Taking inverse Sawi transform of both sides of (24), we get the required solution of (19) with (20) as

$$\begin{bmatrix} \omega(t) = S^{-1}\left\{1 + \frac{1}{p(p^2+1)}\right\} \\ = S^{-1}\{1\} + S^{-1}\left\{\frac{1}{p(p^2+1)}\right\} \end{bmatrix}$$

$$\Rightarrow \omega(t) = t + cost.$$

Problem: 4 Consider the following convolution type Volterra integro-differential equation of first kind

$$\begin{bmatrix} \int_0^t (t-u)^2 \omega(u)du \\ -\frac{1}{12}\int_0^t (t-u)^3 \omega'''(u)du = \frac{t^4}{12} \end{bmatrix} \quad (25)$$

with $\begin{bmatrix} \omega(0) = 0, \\ \omega'(0) = 3, \omega''(0) = 0 \end{bmatrix}$ (26)

Taking Sawi transform of both sides of (25), we have

$$\begin{bmatrix} S\left\{\int_0^t (t-u)^2 \omega(u)du\right\} \\ -\frac{1}{12} S\left\{\int_0^t (t-u)^3 \omega'''(u)du\right\} \\ = \frac{1}{12} S\{t^4\} \end{bmatrix} \quad (27)$$

Applying convolution theorem of Sawi transform on (27), we have

$$\begin{bmatrix} p^2 S\{t^2\} S\{\omega(t)\} \\ -\frac{1}{12} p^2 S\{t^3\} S\{\omega'''(t)\} = 2p^3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2p^3 S\{\omega(t)\} \\ -\frac{1}{12}(6p^4) S\{\omega'''(t)\} = 2p^3 \end{bmatrix} \quad (28)$$

Applying the property "Sawi transform of derivative of functions" on (28), we get

$$\begin{bmatrix} 2p^3 S\{\omega(t)\} \\ -\frac{1}{2}(p^4) \begin{bmatrix} \frac{1}{p^3} S\{\omega(t)\} \\ -\frac{1}{p^4}\omega(0) \\ -\frac{1}{p^3}\omega'(0) \\ -\frac{1}{p^2}\omega''(0) \end{bmatrix} = 2p^3 \end{bmatrix} \quad (29)$$

Now using (26) in (29), we have

$$\begin{bmatrix} 2p^3 S\{\omega(t)\} \\ -\frac{1}{2}(p^4)\left[\frac{1}{p^3} S\{\omega(t)\} - \frac{3}{p^3}\right] = 2p^3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \left[2p^3 - \frac{1}{2}p\right] S\{\omega(t)\} \\ = 2p^3 - \frac{3}{2}p \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \left[\frac{4p^3 - p}{2}\right] S\{\omega(t)\} \\ = 2p^3 - \frac{3}{2}p = \left[\frac{4p^3 - 3p}{2}\right] \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} S\{\omega(t)\} = \left[1 - \frac{2}{4p^2-1}\right] \\ = \left[1 + \frac{2}{1-4p^2}\right] \end{bmatrix} \quad (30)$$

Taking inverse Sawi transform of both sides of (30), we get the required solution of (25) with (26) as

$$\begin{bmatrix} \omega(t) = S^{-1}\left\{1 + \frac{2}{1-4p^2}\right\} \\ = S^{-1}\{1\} + 2S^{-1}\left\{\frac{1}{1-4p^2}\right\} \end{bmatrix}$$

$$\Rightarrow \omega(t) = t + sinh2t.$$

VI. CONCLUSIONS

In this paper, authors successfully discussed the application of Sawi transform for solving convolution type Volterra integro-differential equation of first kind by giving four numerical problems. The results of numerical problems show that the Sawi transform is very useful integral transform for solving convolution type Volterra integro-differential equation of first kind. In future, Sawi transform can be used for solving system of convolution type Volterra integro-differential equations of first kind.

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