Primitive of Second Kind Linear Volterra Integral Equation Using Shehu Transform

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Abstract: **Volterra integral equation occurs widely in diverse areas of applied mathematics, physics, mechanics, statistics, biology, economics and theory of control systems. Many initial value problems associated with ordinary differential equation and partial differential equation can be transformed into a single Volterra integral equation. In this paper, authors determined the primitive of second kind linear Volterra integral equation using Shehu transform. In this work, authors have considered that the kernel of second kind linear Volterra integral equation is a convolution type kernel. Some numerical problems have been considered and solved with the help of Shehu transform for explaining the complete methodology. Results of numerical problems show that Shehu transform is very effective integral transform for determining the primitive of second kind linear Volterra integral equation.**

Keywords: **Volterra integral equation; Shehu transform; Convolution; Inverse Shehu transform.**

I. INTRODUCTION

Integral transforms are highly efficient mathematical techniques for solving many advance problems of science and engineering such as radioactive decay problems, heat conduction problems, problem of motion of a particle under gravity, vibration problems of beam, electric circuit problems and population growth problems. Aggarwal and other scholars [1-8] used different integral transformations (Mahgoub, Aboodh, Shehu, Elzaki, Mohand, Kamal) and determined the analytical solutions of first and second kind Volterra integral equations. Solutions of the problems of Volterra integro-differential equations of second kind are given by Aggarwal et al. [9-11] with the help of Mahgoub, Kamal and Aboodh transformations. In the year 2018, Aggarwal with other scholars [12-13] determined the solutions of linear partial integro-differential equations using Mahgoub and Kamal transformations. Aggarwal et al. [14-20] used Sawi; Mohand; Kamal; Shehu; Elzaki; Laplace and Mahgoub transformations and determined the solutions of population growth and decay problems by the help of their mathematical models. Aggarwal et al. [21-26] defined dualities relations of many advance integral transformations. Comparative studies of Mohand and other integral transformations are given by Aggarwal et al. [27-31]. Aggarwal et al. [32-39] defined Elzaki; Aboodh; Shehu; Sumudu; Mohand; Kamal; Mahgoub and Laplace transformations of error function with applications. The solutions of ordinary differential equations with variable coefficients are given by Aggarwal et al. [40] using Mahgoub transform. Aggarwal et al. [41-45] used different integral transforms and determined the solutions of Abel's integral equations. Aggarwal et al. [46-49] worked on Bessel's functions and determined their Mohand; Aboodh; Mahgoub and Elzaki transformations. Chaudhary et al. [50] gave the connections between Aboodh transform and some useful integral transforms. Aggarwal et al. [51-52] used Elzaki and Kamal transforms for solving linear Volterra integral equations of first kind. Solution of population growth and decay problems was given by Aggarwal et al. [53-54] by using Aboodh and Sadik transformations respectively. Aggarwal and Sharma [55] defined Sadik transform of error function. Application of Sadik transform for handling linear Volterra integro-differential equations of second kind was given by Aggarwal et al. [56]. Aggarwal and Bhatnagar [57] gave the solution of Abel's integral equation using Sadik transform. A comparative study of Mohand and Mahgoub transforms was given by Aggarwal [58]. Aggarwal [59] defined Kamal transform of Bessel's functions. Chauhan and Aggarwal [60] used Laplace transform and solved convolution type linear Volterra integral equation of second kind. Sharma and Aggarwal [61] applied Laplace transform and determined the solution of Abel's integral equation. Laplace transform for the solution of first kind linear Volterra integral equation was given by Aggarwal and Sharma [62]. Mishra et al. [63] defined the relationship between Sumudu and some efficient integral transforms.

The main aim of this paper is to determine the primitive of second kind linear Volterra integral equation using Shehu transform.

II. DEFINITION OF SHEHU TRANSFORM

The Shehu transform of the function $G(t)$ for all $t \geq 0$ is defined as [64]

$$S\{G(t)\} = \int_0^\infty G(t)e^{-\frac{vt}{u}}dt = H(v,u), v > 0, u > 0,$$

where operator S is called the Shehu transform operator.

Fundamental Properties of Shehu Transform

Linearity property [4]: $\begin{bmatrix} If\ S\{G_1(t)\} = H_1(v,u) \\ and\ S\{G_2(t)\} = H_2(v,u) \end{bmatrix}$ then $\begin{bmatrix} S\{aG_1(t) + bG_2(t)\} \\ = a\ H_1(v,u) + bH_2(v,u) \end{bmatrix}$, where a, b are arbitrary constants.

Proof: By the definition of Shehu transform, we have

$$S\{G(t)\} = \int_0^\infty G(t)e^{-\frac{vt}{u}}dt$$

$$\Rightarrow \begin{bmatrix} S\{aG_1(t) + bG_2(t)\} \\ = \int_0^\infty [aG_1(t) + bG_2(t)]e^{-\frac{vt}{u}}dt \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} S\{aG_1(t) + bG_2(t)\} \\ = a\int_0^\infty G_1(t)e^{-\frac{vt}{u}}dt + b\int_0^\infty G_2(t)e^{-\frac{vt}{u}}dt \end{bmatrix}$$

$$\Rightarrow S\{aG_1(t) + bG_2(t)\} = aS\{G_1(t)\} + bS\{G_2(t)\}$$

$$\Rightarrow S\{aG_1(t) + bG_2(t)\} = a\ H_1(v,u) + bH_2(v,u),$$

where a, b are arbitrary constants.

Change of scale property [34]: If Shehu transform of function $G(t)$ is $H(v,u)$ then Shehu transform of function $G(at)$ is given by $\frac{1}{a}H\left(\frac{v}{a}, u\right)$.

Proof: By the definition of Shehu transform, we have

$$S\{G(at)\} = \int_0^\infty G(at)e^{-\frac{vt}{u}}dt$$

Put $at = p \Rightarrow adt = dp$ in above equation, we have

$$S\{G(at)\} = \frac{1}{a}\int_0^\infty G(p)e^{\frac{-vp}{ua}}dp$$

$$\Rightarrow S\{G(at)\} = \frac{1}{a}\left[\int_0^\infty G(p)e^{\frac{-(v/a)p}{u}}dp\right]$$

$$\Rightarrow S\{G(at)\} = \frac{1}{a}H\left(\frac{v}{a}, u\right)$$

Shifting property [34]: If Shehu transform of function $G(t)$ is $H(v,u)$ then Shehu transform of function $e^{at}G(t)$ is given by $H(v - au, u)$.

Proof: By the definition of Shehu transform, we have

$$S\{e^{at}G(t)\} = \int_0^\infty e^{at}G(t)e^{-\frac{vt}{u}}dt$$

$$\Rightarrow S\{e^{at}G(t)\} = \int_0^\infty G(t)e^{-\left(\frac{v}{u}-a\right)t}dt$$

$$= \int_0^\infty G(t)e^{-\left(\frac{v-au}{u}\right)t}dt = H(v - au, u)$$

Shehu transform of the derivatives of the function $G(t)$ [4, 17, 34, 64]:

If $S\{G(t)\} = H(v,u)$ then

$$S\{G'(t)\} = \frac{v}{u}H(v,u) - G(0)$$

$$S\{G''(t)\} = \frac{v^2}{u^2}H(v,u) - \frac{v}{u}G(0) - G'(0)$$

a) $$\begin{bmatrix} S\{G^{(n)}(t)\} \\ = \frac{v^n}{u^n}H(v,u) - \sum_{k=0}^{n-1}\left(\frac{v}{u}\right)^{n-(k+1)}G^{(k)}(0) \end{bmatrix}$$

Shehu transform of integral of a function $G(t)$[34]:

If $S\{G(t)\} = H(v,u)$ then $\begin{bmatrix} S\left\{\int_0^t G(t)dt\right\} \\ = \frac{u}{v}H(v,u) \end{bmatrix}$.

Proof: Let $J(t) = \int_0^t G(t)dt$. Then $J'(t) = G(t)$ and $J(0) = 0$.

Now by the property of Shehu transform of the derivative of function, we have

$$S\{J'(t)\} = \frac{v}{u}S\{J(t)\} - J(0) = \frac{v}{u}S\{J(t)\}$$

$$\Rightarrow S\{J(t)\} = \frac{u}{v}S\{J'(t)\} = \frac{u}{v}\ S\{G(t)\}$$

$$\Rightarrow S\{J(t)\} = \frac{u}{v}H(v,u)$$

$$\Rightarrow S\left\{\int_0^t G(t)dt\right\} = \frac{u}{v}H(v,u)$$

Convolution theorem for Shehu transforms [4, 34]: If Shehu transform of functions $G_1(t)$ and $G_2(t)$ are $H_1(v,u)$ and $H_2(v,u)$ respectively then Shehu transform of their convolution $G_1(t) * G_2(t)$ is given by

$$S\{G_1(t) * G_2(t)\} = S\{G_1(t)\}S\{G_2(t)\}$$

$$\Rightarrow S\{G_1(t) * G_2(t)\} = H_1(v,u)H_2(v,u),$$

where $G_1(t) * G_2(t)$ is defined by

$$\begin{bmatrix} G_1(t) * G_2(t) = \int_0^t G_1(t-x)\ G_2(x)dx \\ = \int_0^t G_1(x)\ G_2(t-x)dx \end{bmatrix}.$$

Proof: By the definition of Shehu transform, we have

$$S\{G_1(t) * G_2(t)\} = \int_0^\infty e^{-\frac{vt}{u}}[G_1(t) * G_2(t)]dt$$

$$\Rightarrow \begin{bmatrix} S\{G_1(t) * G_2(t)\} \\ = \int_0^\infty e^{-\frac{vt}{u}}\left[\int_0^t G_1(t-x)\ G_2(x)dx\right]dt \end{bmatrix}$$

By changing the order of integration, we have

$$\left[\begin{array}{l} S\{G_1(t) * G_2(t)\} \\ = \int_0^\infty G_2(x) \left[\int_x^\infty e^{-\frac{vt}{u}} G_1(t-x)\,dt \right] dx \end{array} \right]$$

Put $t - x = p$ so that $dt = dp$ in above equation, we have

$$\left[\begin{array}{l} S\{G_1(t) * G_2(t)\} \\ = \int_0^\infty G_2(x) \left[\int_0^\infty e^{-\frac{v(p+x)}{u}} G_1(p)\,dp \right] dx \end{array} \right]$$

$$\Rightarrow \left[\begin{array}{l} S\{G_1(t) * G_2(t)\} \\ = \int_0^\infty G_2(x) e^{-\frac{xv}{u}} \left[\int_0^\infty e^{-\frac{pv}{u}} G_1(p)\,dp \right] dx \end{array} \right]$$

$$\Rightarrow S\{G_1(t) * G_2(t)\} = H_1(v,u) H_2(v,u).$$

Table 1 Shehu Transform of Frequently Encountered Functions [4, 17, 34, 64]

S.N.	$G(t)$	$S\{G(t)\} = H(v,u)$
1.	1	$\dfrac{u}{v}$
2.	t	$\left(\dfrac{u}{v}\right)^2$
3.	t^2	$2!\left(\dfrac{u}{v}\right)^3$
4.	$t^n, n \in N$	$n!\left(\dfrac{u}{v}\right)^{n+1}$
5.	$t^n, n > -1$	$\Gamma(n+1)\left(\dfrac{u}{v}\right)^{n+1}$
6.	e^{at}	$\dfrac{u}{v-au}$
7.	$sinat$	$\dfrac{au^2}{(v^2+a^2u^2)}$
8.	$cosat$	$\dfrac{uv}{(v^2+a^2u^2)}$
9.	$sinhat$	$\dfrac{au^2}{(v^2-a^2u^2)}$
10.	$coshat$	$\dfrac{uv}{(v^2-a^2u^2)}$
11.	$J_0(at)$	$\dfrac{u}{\sqrt{(v^2+a^2u^2)}}$
12.	$tsinat$	$\dfrac{2avu^3}{(v^2+a^2u^2)^2}$
13.	$tcosat$	$\dfrac{u^2(v^2-a^2u^2)}{(v^2+a^2u^2)^2}$

III. INVERSE SHEHU TRANSFORM [4]

If $S\{G(t)\} = H(v,u)$ then $G(t)$ is called the inverse Shehu transform of $H(v,u)$ and mathematically it is defined as $G(t) = S^{-1}\{H(v,u)\}$, where S^{-1} is the inverse Shehu transform operator.

Linearity property of inverse Shehu transforms [17]: $\left[\begin{array}{l} If\ S^{-1}\{H_1(v,u)\} = G_1(t) \\ and\ S^{-1}\{H_2(v,u)\} = G_2(t) \end{array} \right]$ *then*

$$\left[\begin{array}{l} S^{-1}\{a\,H_1(v,u) + b\,H_2(v,u)\} \\ = aS^{-1}\{H_1(v,u)\} + bS^{-1}\{H_2(v,u)\} \end{array} \right]$$

$$\Rightarrow \left[\begin{array}{l} S^{-1}\{a\,H_1(v,u) + b\,H_2(v,u)\} \\ = a\,G_1(t) + b\,G_2(t) \end{array} \right], \text{ where } a,b \text{ are arbitrary}$$

constants.

Table 2 Inverse Shehu Transform of Frequently Encountered Functions [17]

S.N.	$H(v,u)$	$G(t) = S^{-1}\{H(v,u)\}$
1.	$\dfrac{u}{v}$	1
2.	$\left(\dfrac{u}{v}\right)^2$	t
3.	$\left(\dfrac{u}{v}\right)^3$	$\dfrac{t^2}{2!}$
4.	$\left(\dfrac{u}{v}\right)^{n+1}, n \in N$	$\dfrac{t^n}{n!}$
5.	$\left(\dfrac{u}{v}\right)^{n+1}, n > -1$	$\dfrac{t^n}{\Gamma(n+1)}$
6.	$\dfrac{u}{v-au}$	e^{at}
7.	$\dfrac{u^2}{(v^2+a^2u^2)}$	$\dfrac{sinat}{a}$
8.	$\dfrac{uv}{(v^2+a^2u^2)}$	$cosat$
9.	$\dfrac{u^2}{(v^2-a^2u^2)}$	$\dfrac{sinhat}{a}$
10.	$\dfrac{uv}{(v^2-a^2u^2)}$	$coshat$
11.	$\dfrac{u}{\sqrt{(v^2+a^2u^2)}}$	$J_0(at)$
12.	$\dfrac{vu^3}{(v^2+a^2u^2)^2}$	$\dfrac{tsinat}{2a}$
13.	$\dfrac{u^2(v^2-a^2u^2)}{(v^2+a^2u^2)^2}$	$tcosat$

IV. PRIMITIVE OF SECOND KIND LINEAR VOLTERRA INTEGRAL EQUATION USING SHEHU TRANSFORM

In this part of the paper, authors determine the primitive of second kind linear Volterra integral equation using Shehu transform. In this work, we have considered that the kernel of second kind linear Volterra integral equation is a convolution type kernel.

Convolution type second kind linear Volterra integral equation is given by

$$y(t) = f(t) + \lambda \int_0^t K(t-x)y(x)dx \qquad (1)$$

where

$$\left. \begin{array}{l} [y(t) = \text{unknown function}], \\ [\lambda = \text{a non} - \text{zero parameter}], \\ \left[\begin{array}{l} K(t-x) = \text{Convolution type kernel} \\ \text{of the integral equation} \end{array} \right], \\ [f(t) = \text{known real} - \text{valued function}] \end{array} \right\}$$

Taking Shehu transform of both sides of (1), we have

$$\left[\begin{array}{l} S\{y(t)\} \\ = S\{f(t)\} + \lambda S\left\{ \int_0^t K(t-x)y(x)dx \right\} \end{array} \right] \qquad (2)$$

Applying convolution theorem of Shehu transform on (2), we have

$$\left[= S\{f(t)\} + \lambda S\{K(t)\}S\{y(t)\}\right] \qquad (3)$$

After simplification (3), we have

$$S\{y(t)\} = \left[\frac{S\{f(t)\}}{1 - \lambda S\{K(t)\}}\right] \qquad (4)$$

Now taking inverse Shehu transform on both sides of (4), we get

$$y(t) = S^{-1}\left\{\left[\frac{S\{f(t)\}}{1 - \lambda S\{K(t)\}}\right]\right\} \qquad (5)$$

which is the required solution of (1).

V. NUMERICAL PROBLEMS

In this part of the paper, some numerical problems have been considered for explaining the complete methodology.

Problem: 1 Consider the following second kind linear Volterra integral equation

$$\left[y(t) = sint - 2\int_0^t cos(t-x)y(x)dx\right] \qquad (6)$$

Taking Shehu transform of both sides of (6), we have

$$\left[= S\{sint\} - 2S\left\{\int_0^t cos(t-x)y(x)dx\right\}\right] \qquad (7)$$

Applying convolution theorem of Shehu transform on (7), we have

$$[S\{y(t)\} = S\{sint\} - 2S\{cost\}S\{y(t)\}]$$

$$\Rightarrow \left[S\{y(t)\} = \frac{u^2}{(v^2+u^2)} - 2\frac{uv}{(v^2+u^2)}S\{y(t)\}\right] \qquad (8)$$

After simplification (8), we have

$$S\{y(t)\} = \left[\frac{u^2}{(u+v)^2}\right] \qquad (9)$$

Now taking inverse Shehu transform on both sides of (9), we get

$$y(t) = S^{-1}\left\{\left[\frac{u^2}{(u+v)^2}\right]\right\} = te^{-t} \qquad (10)$$

which is the required solution of (6).

Problem: 2 Consider the following second kind linear Volterra integral equation

$$\left[y(t) = cost + sint - \int_0^t y(x)\,dx\right] \qquad (11)$$

Taking Shehu transform of both sides of (11), we have

$$\left[= S\{cost\} + S\{sint\} - S\left\{\int_0^t y(x)\,dx\right\}\right] \qquad (12)$$

Applying convolution theorem of Shehu transform on (12), we have

$$\left[S\{y(t)\} = \frac{uv}{(v^2+u^2)} + \frac{u^2}{(v^2+u^2)} - S\{1\}S\{y(t)\}\right] \Rightarrow \left[S\{y(t)\} = \frac{uv}{(v^2+u^2)} + \frac{u^2}{(v^2+u^2)} - \frac{u}{v}S\{y(t)\}\right] \qquad (13)$$

After simplification (13), we have

$$S\{y(t)\} = \left[\frac{uv}{(v^2+u^2)}\right] \qquad (14)$$

Now taking inverse Shehu transform on both sides of (14), we get

$$y(t) = S^{-1}\left\{\frac{uv}{(v^2+u^2)}\right\} = cost \qquad (15)$$

which is the required solution of (11).

Problem: 3 Consider the following second kind linear Volterra integral equation

$$\left[y(t) = t - \int_0^t (t-x)y(x)\,dx\right] \qquad (16)$$

Taking Shehu transform of both sides of (16), we have

$$\left[S\{y(t)\} = S\{t\} - S\left\{\int_0^t (t-x)y(x)\,dx\right\}\right] \qquad (17)$$

Applying convolution theorem of Shehu transform on (17), we have

$$[S\{y(t)\} = S\{t\} - S\{t\}S\{y(t)\}]$$

$$\Rightarrow \left[S\{y(t)\} = \left(\frac{u}{v}\right)^2 - \left(\frac{u}{v}\right)^2 S\{y(t)\}\right] \qquad (18)$$

After simplification (18), we have

$$S\{y(t)\} = \left[\frac{u^2}{(v^2+u^2)}\right] \qquad (19)$$

Now taking inverse Shehu transform on both sides of (19), we get

$$y(t) = S^{-1}\left\{\frac{u^2}{(v^2+u^2)}\right\} = sint \qquad (20)$$

which is the required solution of (16).

Problem: 4 Consider the following second kind linear Volterra integral equation

$$\left[y(t) = 1 - t - \int_0^t (t-x)y(x)\,dx\right] \qquad (21)$$

Taking Shehu transform of both sides of (21), we have

$$\left[= S\{1\} - S\{t\} - S\left\{\int_0^t (t-x)y(x)\,dx\right\}\right] \qquad (22)$$

Applying convolution theorem of Shehu transform on (22), we have

$$[S\{y(t)\} = S\{1\} - S\{t\} - S\{t\}S\{y(t)\}]$$

$$\Rightarrow \left[S\{y(t)\} = \left(\frac{u}{v}\right) - \left(\frac{u}{v}\right)^2 - \left(\frac{u}{v}\right)^2 S\{y(t)\} \right] \qquad (23)$$

After simplification (23), we have

$$S\{y(t)\} = \left[\frac{uv}{(v^2+u^2)} - \frac{u^2}{(v^2+u^2)} \right] \qquad (24)$$

Now taking inverse Shehu transform on both sides of (24), we get

$$y(t) = S^{-1}\left\{ \frac{uv}{(v^2+u^2)} - \frac{u^2}{(v^2+u^2)} \right\}$$

$$= S^{-1}\left\{ \frac{uv}{(v^2+u^2)} \right\} - S^{-1}\left\{ \frac{u^2}{(v^2+u^2)} \right\}$$

$$= cost - sint \qquad (25)$$

which is the required solution of (21).

Problem: 5 Consider the following second kind linear Volterra integral equation

$$\left[y(t) = sint + 2\int_0^t e^{(t-x)}y(x)\,dx \right] \qquad (26)$$

Taking Shehu transform of both sides of (26), we have

$$\left[\begin{array}{c} S\{y(t)\} \\ = S\{sint\} + 2S\left\{ \int_0^t e^{(t-x)}y(x)\,dx \right\} \end{array} \right] \qquad (27)$$

Applying convolution theorem of Shehu transform on (27), we have

$$[S\{y(t)\} = S\{sint\} + 2S\{e^t\}S\{y(t)\}]$$

$$\Rightarrow \left[S\{y(t)\} = \frac{u^2}{(v^2+u^2)} + 2\left(\frac{u}{v-u}\right)S\{y(t)\} \right] \qquad (28)$$

After simplification (28), we have

$$S\{y(t)\} = \frac{1}{5}\left[\frac{u}{v-3u} \right] - \frac{1}{5}\left[\frac{uv}{(v^2+u^2)} \right] + \frac{2}{5}\left[\frac{u^2}{(v^2+u^2)} \right] \qquad (29)$$

Now taking inverse Shehu transform on both sides of (29), we get

$$y(t) = S^{-1}\left\{ \begin{array}{c} \frac{1}{5}\left[\frac{u}{v-3u} \right] - \frac{1}{5}\left[\frac{uv}{(v^2+u^2)} \right] \\ + \frac{2}{5}\left[\frac{u^2}{(v^2+u^2)} \right] \end{array} \right\}$$

$$= \left[\begin{array}{c} \frac{1}{5}S^{-1}\left\{ \frac{u}{v-3u} \right\} - \frac{1}{5}S^{-1}\left\{ \frac{uv}{(v^2+u^2)} \right\} \\ + \frac{2}{5}S^{-1}\left\{ \frac{u^2}{(v^2+u^2)} \right\} \end{array} \right]$$

$$= \frac{1}{5}e^{3t} - \frac{1}{5}cost + \frac{2}{5}sint \qquad (30)$$

which is the required solution of (26).

VI. CONCLUSIONS

In this paper, authors successfully determined the primitive of second kind linear Volterra integral equation using Shehu transform and five numerical problems solved in numerical problem session of the paper for explaining the complete methodology. The results of numerical problems show that the Shehu transform is very useful integral transform for handling the problem of determining the primitive of second kind linear Volterra integral equation. In future, Shehu transform can be used for determining the primitive of system of second kind linear Volterra integral equations.

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