

The Use of Box–Jenkins Method to Predict the Six Sigma Level for Kut Technical Institute Examination Results

Alla Hussein Omran Ansaf

Assistant Professor, Kut Technical Institute, Middle Technical University, Baghdad, Iraq

Abstract: In this study, ARIMA model was used to forecast six sigma level of the results of the examinations for the students of Kut Technical Institute. The time series data in our study was level of six sigma for the results of the examinations of the students in the Institute from academic year 2003-2004 to academic year 2017-2018. The researcher used Box-Jenkins methodology and noted there is a Trend Variation. To remove the non-stationary, the first difference was taken. Using statistical software EVIEWS 9, ARIMA (1,1,1) was the best model chosen based on the SSE, adj R², AIC, BIC, and H-Q criteria. A validation check for this model was performed on residuals series, were found white noise, normally distributed, and equal in variance. The predicted results from the selected model were compared with the original data to determine prediction precision. It found that the selected model predicted six sigma level with acceptable accuracy. These results will provide the Institute's managers with decisions on how to upgrade six sigma level.

Keywords: Six Sigma, DAMIC, Time Series Analysis, ARIMA, Correlogram.

I. INTRODUCTION

Higher education is one of the most important areas of life because of the direct relationship between the quality of higher education and the growth of the cultural and economic community. It is not possible to achieve a distinct level of quality of higher education without the concerted efforts of all employees, students, labor market, and society. Six sigma is one of the best quality assurance methods that achieve the best quality with the least defects to reach advanced levels in the world rankings of universities. In order to implement six sigma, there must be a broad database of data and information that guides decision makers to make decisions about the learning process.

Box-Jenkins method is one of the most prominent models used in time series analysis (seasonal and non-seasonal), which were formulated by statisticians Box-Jenkins.

The aims of this study is to shed light on the six sigma methodology and the possibility of applying its principles to the results of the examinations for the students of the Kut Technical Institute / the Middle Technical University, which is one of the institutions of higher education in Iraq, and then

diagnosing the best ARIMA model that can be used to predict the level of six sigma for the next years.

The importance of this research comes through the introduction of six sigma methodology and its role in knowledge of the quality of the results of examinations for students of Kut Technical Institute, and thus enables the deanship of the Institute to address errors and failures in the educational process, and then develop a standard model to predict six sigma level for next years.

II. SIX SIGMA APPROACH

Six sigma is a statistical procedure that determines to what extent a specific process deviates from perfection (Vivekananthamoorthy & Sankar, 2011). The main focus of six sigma is to reduce defects and differences in the process and get a consistent and predictable process (Montgomery & Woodall 2008). Six sigma methodologies are disseminated across various companies from small businesses to prominent companies. It was founded for Motorola early 1986 by a specialist engineer called Bill Smith. Smith, the founder of Six Sigma, studied differences in the results and identified it in the internal processes of the company's activity and attributed these differences to errors and highlighted the possibility of improvement system performance by reducing errors (Ali & Ahmed 2016). Six sigma produces a product with a defect of 3.4 per million opportunities translated into quality or production of 99.9997 percent.

DMAIC procedure is one of the best six sigma approaches, it includes five phases; define, measure, analysis, improve and control (Antony & Banuelas, 2001; Sodhi et al., 2017). Define phase, is the stage of identifying the problem and its importance. Measure phase, is the stage of data collection of the problem and thus can identify the statistical tools used to reach the required quality. Analysis phase is the stage of we analyze all data to find the causes of the problem and identify the variables that cause the defects. Improve phase, a set of measures aimed at improving performance, improving the level of service, developing possible alternatives to solve the problem, and then choosing the appropriate solution and ensuring that the proposed solution is the best. Control phase, where at this stage ensure that the defects are not repeated to ensure compliance with quality.

To achieve the goal of this study, the data were collected from the results of the final exams for all the students of the Institute for the academic year 2003/2004 to 2017/2018. Data were collected according to the concept of six sigma such that:

- The student is considered one unit.
- The failed student is considered a defect unit.
- Defect per unit (DPU) is the sum of the defects of (n) of a defect unit divided by the total number of units.

$$DPU = \frac{\text{number of defect}}{\text{number of units}} \quad (1)$$

- The number of subjects in which the students failed considers defect opportunity.

- Defect per Opportunity (DPO)

$$DPO = \frac{\text{Number of defect}}{\text{number of opportunities}} \quad (2)$$

- Defects per million opportunities (DPMO)

$$DPMO = DPO \times 10^6 \quad (3)$$

$$\text{g. Sigma Equality Level} = 0.8406 + \sqrt{29.37 - 2.221 \ln(DPMO)} \quad (4)$$

III. BOX- JENKINS MODEL

The Box-Jenkins method was circulated by George Box and Gwilym Jenkins in 1970 (Box, et. al., 2015). It is widely used by researchers because it gives ideal expectations if all theoretical tests are performed. ARMA model is considered one of the best time series models, it is produced by a combination a model of autoregressive AR(P) and the model of moving-average MA(q).

Autoregressive AR (P) is formulated as follows:-

$$y_t = \alpha + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots + \phi_p y_{t-p} + \varepsilon_t \quad (5)$$

The model of moving average MA(q) is formulated as follows:

$$y_t = \alpha - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \dots - \theta_q \varepsilon_{t-q} + \varepsilon_t \quad (6)$$

Whereas:

y_t : "The actual value, $t = 1, 2, \dots, n$ "

ε_t : "Error term at time t"

ϕ : "Parameters of autoregressive model"

θ : "Parameters of moving average model"

α : "The intercept of the model"

p: "The order of autoregressive model"

q: "The order of moving average model"

After combining the above two models the ARMA (p, q) model is obtained, which is formulated as follows:

$$y_t = \alpha + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots + \phi_p y_{t-p} - \phi_1 \varepsilon_{t-1} - \phi_2 \varepsilon_{t-2} - \dots - \phi_q \varepsilon_{t-q} + \varepsilon_t \quad (7)$$

Because many of the time series are non-stationary, G – Box and G- Jenkins have proposed a new model by taking a differencing in time series data, this model named autoregressive integrated moving average ARIMA (p, d, q), such that d represent the number of differences required to obtain the stationary. There are four stages of applying the methodology of Box – Jenkins.

Stage 1: Identification

At this stage, the stationary of the time series is confirmed, that is, it should have a constant mean and variance through time, and no seasonality. Also, at this stage the appropriate values of p, q are found out by correlogram.

For the purpose of determining the stationary of the time series, the following tests are done:

- The original values are plotted as well as correlogram to obtain a good idea of whether or not pairs of data show auto-correlation. The data are non-stationary if ACF dies down slowly (Box, et. al., 2015).
- Ljung-Box Q-statistic test is use to test the null hypothesis that there is no autocorrelation. It is also used to test if the series is white noise or not. The test statistics is:

$$Q = n(n+2) \sum_{k=1}^h \frac{\hat{\rho}_k^2}{n-k} \quad (8)$$

Where $\hat{\rho}_k$ is autocorrelation coefficient (for lag $k=1$ to h), n number of observations. For the significance level α (Ljung& Box ,1978), the critical area to reject the hypothesis is:

$$Q > \chi^2_{(\alpha, h)}$$

- Unit root test use to exam if data are stationary or not using Augmented Dickey-Fuller (ADF). When the critical value is greater than ADF value, we cannot reject the null hypothesis, (H_0 : data is not stationary), (Brooks, C. 2014).
- Differencing is one method to remove non-stationarity, Natural Logarithmic and Square Root transformation are popular methods to remove variance not stationary (Hyndman, R.J., and Athanasopoulos, G. 2018).

Stage 2: Estimation

After determining the appropriate model, its characteristics are estimated by using one of the estimation methods which varies according to the model used. The most procedures used

Non-Linear least squares method (NLS), and Maximum Likelihood Method (ML). The appropriate model should have most significant coefficients (Lawrence & Paul, 1978).

Stage 3: Diagnostic checking

The following tests are conducted to examine the fitness of the model:

- Using t- test to check the significance of parameters, if the P- value of the test greater than the significant level of 5%, the model is accepted.
- Plotting ACF and PACF of the residual to ensure that the residuals are independent and constant in mean and variance over time.
- Testing the normality distribution of the residuals by plotting the normal probability of the residuals and using Jarque - Bera test (Jarque, & Bera, 1980).
- Testing randomness of the residuals, if the residuals fell between the 95% of the confidence level of correlograms of ACF and PACF, this indicates that the residuals are random.
- Testing ACF using the Ljung-Box test, if the residuals pointing to no auto-correlation, this is indicate to a good fitted model.
- Test of heteroscedasticity. It is very important to verify that the series of errors are equal in variance. If the residuals in the equations do not have constant variance, they are said to be heteroskedastic. There are a number of tests for non-verification of excess heterogeneity such as; ARCH LM test (Engle 1982), and White's test (White's, 1980).
- Test of the Breusch-Godfrey Serial Correlation. It is examined whether correlation was not included in the proposed model structure, which if present, will mean that incorrect conclusions will be drawn from other tests. Because the test is based on the idea of the Lagrange multiplier test, sometimes referred to as the LM for serial correlation testing (Asteriou&Hall, 2011).

Some models can pass all the above tests. In order to choose the best model, it is selected according to the following criteria:

- A Kaike Information Criterion (AIC). It is given by the following equation (Akaike, 1974).

$$AIC = -2Ln(\hat{\sigma}_\varepsilon^2) + 2 \frac{(p + q)}{n} \quad (9)$$

- Schwarz Bayes information criterion. Schwarz suggested the following test:

$$BIC = -2n Ln(\hat{\sigma}_\varepsilon^2) + \frac{(p + q)}{n} Ln(n) \quad (10)$$

Where $\hat{\sigma}_\varepsilon^2$ is the error variance (Schwarz, 1978).

- Hannan - Quinn method (H - Q), given as follows (Hannan & Quinn 1979).

$$H - Q = -2n Ln(\hat{\sigma}_\varepsilon^2) + 2 \frac{(p + q)}{n} (LnLn(n)) \quad (11)$$

- Adjusted R^2 , it gives an idea of how many data points fall within the line of the regression equation and explain the ratio of the variation find only by independent variables that actually affect the dependent variable. It is given as follows (Theil, Henri 1961).

$$adj R^2 = 1 - \frac{(1 - R^2)(n - 1)}{n - k - 1} \quad (12)$$

- Sum of squared errors (SSE), it is a measure of the discrepancy between the data and the estimation model. It is given as following equation (Draper& Smith1998).

$$SSE = S_{yy} (1 - r^2), \text{ where } S_{yy} = \sum_{i=1}^n (\bar{y} - y_i)^2 \quad (13)$$

The model with the lowest of SSE, AIC, BIC, H-Q criteria, and the highest value of adjusted R^2 is selected.

Stage 4: Prediction

After estimating the parameters of the best model, this model is used in prediction by replacing the current and past values of the dependent variable y_t and the residual as estimated value for the first predicated value. The prediction is done sequentially, i.e. the first predictive value is used to predict the second value, and so on.

IV. RESULTS

Using Excel Microsoft Office, the level of six sigma was calculated for all the students of the Institute for the academic years under study, which has 15 observations and presented in table I.

By using the statistical program Eviews 9 the results were obtained to achieve the research objectives, where the mathematical mean of the time series was equal to 1.6744 and a standard deviation of 0.2444. The original data are plotted to identify its initial properties as in Fig. 1. It is found there is a Trend Variation through this mean, that is, the time series is not stationary over time.

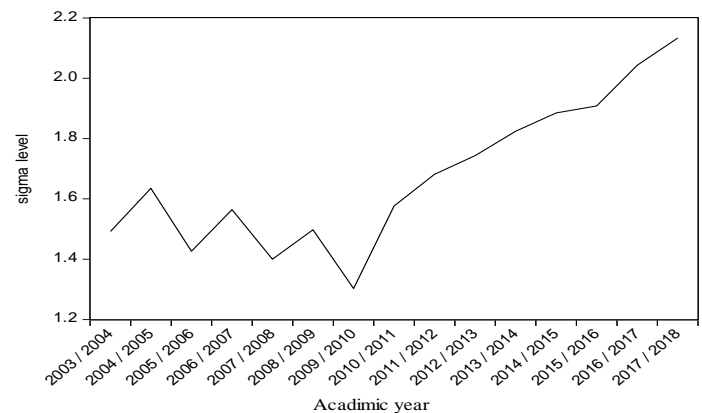


Fig. 1: Level of six sigma from academic year 2003/2004 to 2017/2018

Table I: Six Sigma Level for Different Academic Year

Academic year	No. units	No. defect	No. opp-ortunities	DPU	DPO	DPMO	Sigma Level
2003 / 2004	1384	197	431	0.14234	0.45707	457077	1.49220
2004 / 2005	969	147	353	0.15170	0.416431	416431	1.63522
2005 / 2006	886	174	367	0.19639	0.474114	474114	1.42652
2006 / 2007	1198	184	421	0.15359	0.437055	437055	1.56452
2007 / 2008	1256	261	543	0.20780	0.480663	480663	1.39991
2008 / 2009	1464	354	777	0.24180	0.455598	455598	1.49770
2009 / 2010	1945	285	567	0.14653	0.502646	502646	1.30267
2010 / 2011	2172	428	987	0.19705	0.433637	433637	1.57647
2011 / 2012	1930	524	1302	0.27150	0.402458	402458	1.68157
2012 / 2013	1573	455	1187	0.28926	0.383319	383319	1.74362
2013 / 2014	1682	409	1143	0.24316	0.357830	357830	1.82460
2014 / 2015	1680	322	951	0.19167	0.338591	338591	1.88511
2015 / 2016	1859	202	610	0.10866	0.331148	331148	1.90849
2016 / 2017	2499	238	825	0.09524	0.288485	288485	2.04339
2017 / 2018	3386	183	702	0.05405	0.260684	260684	2.13357

To ensure that the time series was not stationary, ACF and PACF were plotted as show in Fig. 2. It is seen that ACF does not decrease quickly to zero which is indicate that the time series is not stationary. Also, it was observed that a number of coefficients were not within confidence limit at 95%, $p_r \left\{ -1.96 \left[\frac{1}{\sqrt{n}} \right] \leq p_k \leq +1.96 \left[\frac{1}{\sqrt{n}} \right] \right\} = \pm 0.506$ and this was confirmed by Q statistic test.

$(Q = 46.450) > (\chi^2_{(0.05, 12)} = 21.03)$, thus it is concluded that the original data were non-stationary.

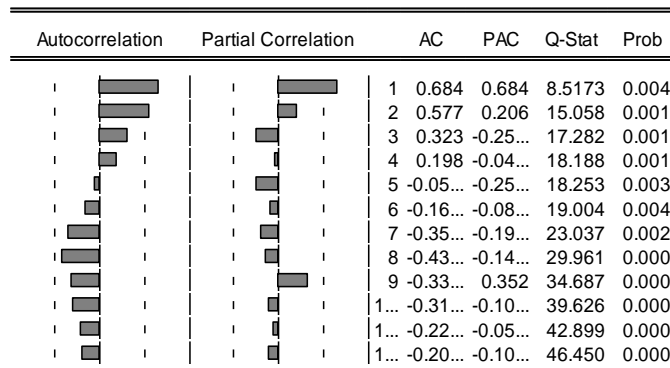


Fig. 2: Correlogram of original data

After that, the unit root of the original data is tested by using Augmented Dickey-Fuller (ADF) as in the table II.

Table II : Unit Root Test of Original Data

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-1.953998	0.5744
Test critical values:		
1% level	-4.800080	
5% level	-3.791172	
10% level	-3.342253	

Based on the table II, it shows that the absolute value at $\alpha = 5\%$ is (3.791172) which is greater than the absolute value of the statistic t of the ADF test (1.953998), i.e. the hypothesis of a unit root was accepted and, this indicates that the data is non-stationary. Therefore, the first difference of the series was taken and plotted as in Fig. 3, where it is observed that the time series has become stable.

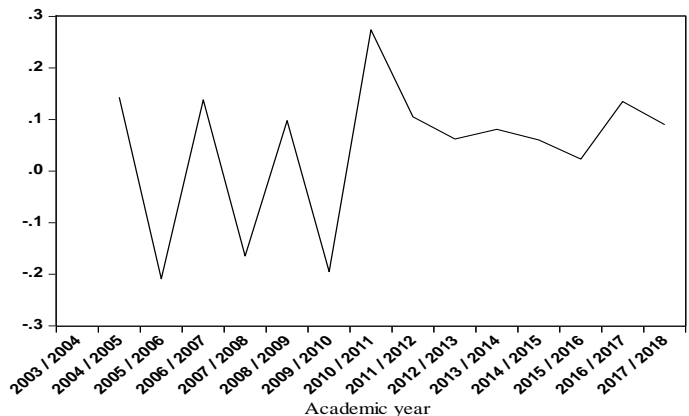


Fig. 3: First difference for the level of six sigma

This is confirmed by the plotting the correlogram of the first difference of data as in Fig. 4.

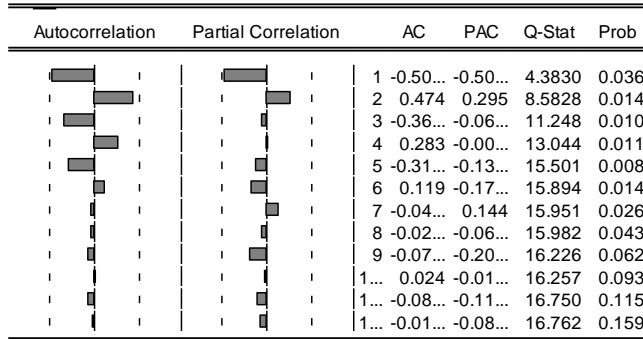


Fig. 4: Correlogram of first difference

Because the plots decrease to the zero, it is concluded that adjusted data are stationary. Also, it was observed that all the coefficients were within the confidence limits at 95%, and this was confirmed by Q statistic test ($Q= 16.762 < (\chi^2_{(0.05, 12)} = 21.03)$), thus it is concluded that the adjusted data are stationary.

Also the unit root of the first difference of data was tested by using Augmented Dickey-Fuller (ADF) as shown in table III.

Table III : Unit Root Test Of First Difference

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-8.05082...	0.0001
Test critical values: 1% level	-4.886426	
5% level	-3.828975	
10% level	-3.362984	

Based on table III, it shows that the absolute value at $\alpha = 5\%$, critical value is (3.828975) which is less than the absolute value of the ADF test statistic (8.05082). This means that the first difference of our time series is stationary.

From the correlogram of adjusted data, ACF has a large spike at first lag but wobbling, and PACF has a large spike at first lag. So three models have been temporarily selected; ARIMA (1,1,0), ARIMA (0,1,1), and ARIMA (1,1,1) as shown in table IV below.

Table IV. Measures of Accuracy For Alternative Arima Models

Model	SSE	Adjusted R ²	AIC	BIC	H-Q
ARIMA(1,1,0)	0.12480 5	0.206928	1.168870	1.07757 6	1.17732 1
ARIMA(0,1,1)	0.13344 7	0.093303	1.049572	0.95827 9	1.05802 3
ARIMA(1,1,1)	0.11707 9	0.302083	0.499139	0.36219 8	0.51181 5

The ARIMA (1,1,1) model has the lowest of SSE, AIC, BIC, H-Q criteria, and the highest value of adjusted R², thus it is concluded that this model is the best.

5. Parameters Estimation

By applying the method of Generalized Least Squares (GLS) method, the following results were obtained as shown in table V below.

Table V : Estimated Parameters For Arima(1,1,1)

Type	coefficient	Standard Error of coefficient	T-Statistic	P- value
Constant	0.041517	0.026862	1.545558	0.1505
AR(1)	-0.999998	0.004397	-227.4219	0.0000
MA(1)	0.702377	0.214569	3.273440	0.0074

From the above table, it is found that the parameters of the model are statistically significant. Since the parameter of AR (1) and MA (1) are significant, then the ARIMA (1,1,1) model can be included as a possible model.

6. Checking the efficiency of the model

In order to verifying the efficiency of the specified model to predicting, the following work has been done:

- For the randomized residuals, the correlogram was plotted as in Fig. 5, where they found that ACF and PACF to be within the limits of confidence ± 0.506 .
- Qstastic test is done, ($Q= 4.9408 < (\chi^2_{(0.05, 12)} = 21.03)$), which meant that all residuals are random and white noise and that the model is pretty well statistically.

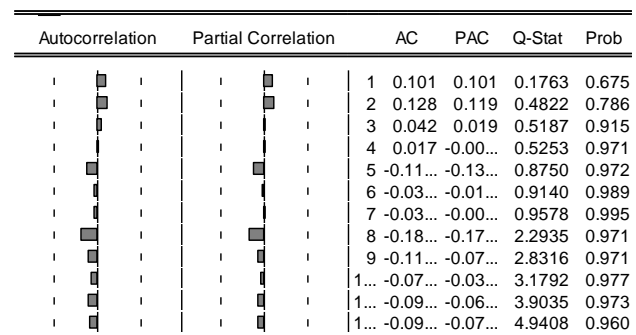


Fig. 5: Correlogram of Residuals

- Jarque - Bera test for normality test whether the residuals are normally distributed as illustrated in Fig. 6. The graph shows, P-value associated with the test equal 0.502981 which is higher than 0.05, thus the null hypothesis for the normal distribution is accepted.

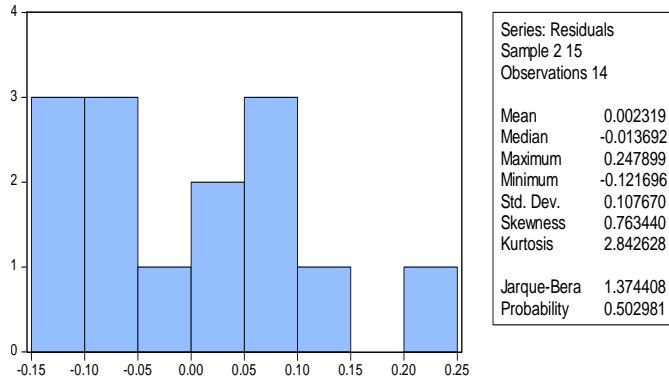


Fig. 6: Testing the Normality of Residuals

d. Table VI shows the Heteroscedasticity tests. It is seen that F- statistic for ARCH test is 0.15629 and the p-value is 0.7001, thus we cannot reject the null hypothesis which assumes that the residuals are homoscedastic. Also the same conclusion from White test is obtained.

Table VI: The Heteroscedasticity Tests

a: ARCH			
F-statistic	0.15629...	Prob. F(1,11)	0.7001
Obs*R-squared	0.18212...	Prob. Chi-Square(1)	0.6696
b: White			
F-statistic	0.98485...	Prob. F(9,4)	0.5510
Obs*R-squared	9.64665...	Prob. Chi-Square(9)	0.3798
Scaled explained S...	5.69022...	Prob. Chi-Square(9)	0.7705

Table VII shows test of the Breusch-Godfrey Serial Correlation. It is seen that F- statistic for this test is 0.224754 and the p-value is 0.8031. According to this test, the null hypothesis is accepted (Residuals are not correlated).

Table VII Breusch-Godfrey Serial Correlation Test

F-statistic	0.224754	Prob. F(2,9)	0.8031
Obs*R-squared	0.659312	Prob. Chi-Square(2)	0.7192

From above tests of the residuals results, the validity of the estimated ARIMA (1,1,1) model is confirmed to represent the time series and can be used in the prediction process.

7. Forecasting

Fig. 7 represent the actual and fitted data for the expected Sigma level according to the ARIMA model (1,1,1). It is seen that the fitted values getting a close to the actual values. The residuals are close to zero form the academic year 2011/ 2012 of the series, which means that the forecasting will be accurate.

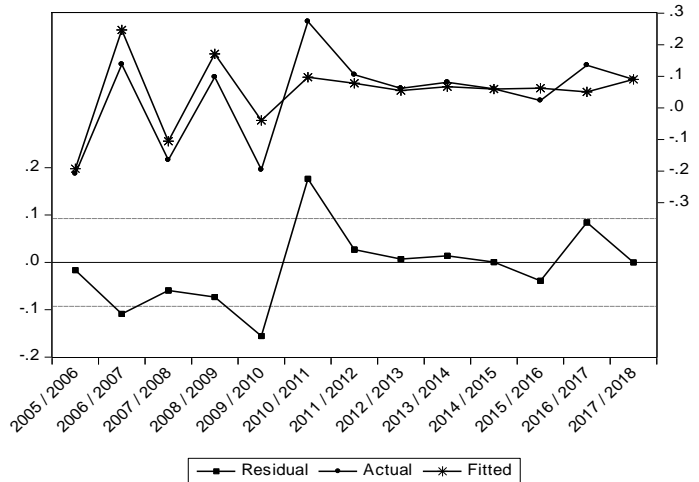


Fig. 7: Comparison between actual and fitted data for ARIMA(1,1,1) model

Thus, the estimated equation is:

$$y_t = 0.041517 - 0.999998y_{t-1} + 0.702377 \epsilon_{t-1} + \epsilon_t$$

According to the proposed model, the six sigma level has been predicted for the next six years as shown in the table VIII.

Table VIII: Six Sigma Level Forecasting For Next Six Years

Academic year	forecast	95% confidence Limits	
		Lower	Upper
2018/2019	2.19631	1.96751	2.42511
2019/2020	2.21670	1.93639	2.49700
2020/2021	2.27933	1.91761	2.64104
2021/2022	2.29982	1.90344	2.69621
2022/2023	2.36234	1.90484	2.81984
2023/2024	2.38295	1.89752	2.86838

IV. CONCLUSIONS

The results obtained can be summarized as follows:

- There is a trend variation in the time series that was removed by taking the first difference.
- The best model was chosen from among the possible models which minimizes the SSE, AIC, BIC, and H-Q criteria and biggest adjusted R^2 . The suitability of the model proposed was examined statistically through a number of tests such that: significance of the estimated parameters, examine ACF and PACF of the residuals to ensure that the residuals are independent and random, testing the normality distribution of the residuals, and using ARCH LM test and White's test to check that the residuals are equal in variance.

- c. It was found that the appropriate and efficient model for the representation of time series data is ARIMA (1,1,1). This model has met all the requirements of the tests.
- d. The six sigma level was predicted and found that the expected level consistent with the original time series and indicates a decrease from the required level.
- e. This study will help academics in the higher education sector to improve the quality of higher education by examining the results of exams to know the level of six sigma and predict it for the coming years, also find out the reasons that led to the reduction of the level of six sigma.

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