On The Existence of Solution of Diophantine Equation $181^x + 199^y = z^2$

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Abstract: **In this article, authors discussed the existence of solution of Diophantine equation $181^x + 199^y = z^2$, where x, y, z are non-negative integers. Results show that the consider Diophantine equation of study has no non-negative integer solution.**

Keywords**: Prime number; Diophantine equation; Solution, Integers.**

Mathematics Subject Classification: 11D61, 11D72, 11D45.

I. INTRODUCTION

Diophantine equations are those equations of theory of numbers which are to be solved in integers. Diophantine equations have many important applications in algebra, analytical geometry and trigonometry [4, 6]. These equations give us an idea to prove the existence of irrational numbers. Acu [1] studied the Diophantine equation $2^x + 5^y = z^2$ and proved that $\{x = 3, y = 0, z = 3\}$ and $\{x = 2, y = 1, z = 3\}$ are the solutions of this equation. Kumar et al. [2] considered the non-linear Diophantine equations $61^x + 67^y = z^2$ and $67^x + 73^y = z^2$. They showed that these equations have no non-negative integer solution. Kumar et al. [3] studied the non-linear Diophantine equations $31^x + 41^y = z^2$ and $61^x + 71^y = z^2$. They determined that these equations have no non-negative integer solution. Rabago [5] discussed the open problem given by B. Sroysang. He showed that the Diophantine equation $8^x + p^y = z^2$, where x, y, z are positive integers has only three solutions namely $\{x = 1, y = 1, z = 5\}$, $\{x = 2, y = 1, z = 9\}$ and $\{x = 3, y = 1, z = 23\}$ for $p = 17$. The Diophantine equations $8^x + 19^y = z^2$ and $8^x + 13^y = z^2$ were studied by Sroysang [7-8]. He proved that these equations have a unique non-negative integer solution namely $\{x = 1, y = 0, z = 3\}$. Sroysang [9] proved that the Diophantine equation $31^x + 32^y = z^2$ has no non-negative integer solution.

The main aim of this article is to discuss the existence of solution of Diophantine equation $181^x + 199^y = z^2$, where x, y, z are non-negative integers.

II. PRELIMINARIES

Lemma: 1 The Diophantine equation $181^x + 1 = z^2$, *where* x, z *are non-negative integers, has no solution in non-negative integers.*

Proof: Since 181 is an odd prime so 181^x is an odd number for all non-negative integer x.

$\Rightarrow 181^x + 1 = z^2$ is an even number for all non-negative integer x.

$\Rightarrow z$ is an even number.

$\Rightarrow z^2 \equiv 0(mod3)$ or $z^2 \equiv 1(mod3)$ (1)

Now, $181 \equiv 1(mod3)$

$\Rightarrow 181^x \equiv 1(mod3)$, for all non-negative integer x

$\Rightarrow 181^x + 1 \equiv 2(mod3)$, for all non-negative integer x

$\Rightarrow z^2 \equiv 2(mod3)$ (2)

Equation (2) contradicts equation (1). Hence Diophantine equation $181^x + 1 = z^2$ has no non-negative integer solution.

Lemma: 2 The Diophantine equation $199^y + 1 = z^2$, *where* y, z *are for all non-negative integers, has no solution in non-negative integers.*

Proof: Since 199 is an odd prime so 199^y is an odd number for all non-negative integer y.

$\Rightarrow 199^y + 1 = z^2$ is an even number for all non-negative integer y

$\Rightarrow z$ is an even number

$\Rightarrow z^2 \equiv 0(mod3)$ or $z^2 \equiv 1(mod3)$ (3)

Now, $199 \equiv 1(mod3)$

$\Rightarrow 199^y \equiv 1(mod3)$, for all non-negative integer y

$\Rightarrow 199^y + 1 \equiv 2(mod3)$, for all non-negative integer y

$\Rightarrow z^2 \equiv 2(mod3)$ (4)

Equation (4) contradicts equation (3). Hence Diophantine equation $199^y + 1 = z^2$ has no non-negative integer solution.

Main Theorem: The Diophantine equation $181^x + 199^y = z^2$, where x, y, z are non-negative integers, has no solution in non-negative integers.

Proof: There are three cases:

Case: 1 If $x = 0$ then the Diophantine equation $181^x + 199^y = z^2$ becomes

$1 + 199^y = z^2$, which has no non-negative integer solution by lemma 2.

Case: 2 If $y = 0$ then the Diophantine equation $181^x + 199^y = z^2$ becomes $181^x + 1 = z^2$, which has no non-negative integer solution by lemma 1.

Case: 3 If x, y are positive integers, then $181^x, 199^y$ are odd numbers.

$\Rightarrow 181^x + 199^y = z^2$ is an even number

$\Rightarrow z$ is an even number

$\Rightarrow z^2 \equiv 0(mod3)\ or\ z^2 \equiv 1(mod3)$ (5)

Now, $181 \equiv 1(mod3)$

$\Rightarrow 181^x \equiv 1(mod3)\ and\ 199 \equiv 1(mod3)$

$\Rightarrow 181^x \equiv 1(mod3)\ and\ 199^y \equiv 1(mod3)$

$\Rightarrow 181^x + 199^y \equiv 2(mod3)$

$\Rightarrow z^2 \equiv 2(mod3)$ (6)

Equation (6) contradicts equation (5). Hence Diophantine equation $181^x + 199^y = z^2$ has no non-negative integer solution.

III. CONCLUSION

In this article, authors successfully discussed the solution of Diophantine equation $181^x + 199^y = z^2$, where x, y, z are non-negative integers and determined that this equation has no non-negative integer solution.

CONFLICT OF INTERESTS

Authors state that this paper has no conflict of interest.

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