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Exploring the Dynamics of Turbulent Flows: Mathematical Models and Stability in Curved Channels

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ABSTRACT

This study explores the dynamics of turbulent flows in curved channels, with a focus on developing and validating mathematical models that accurately represent flow behaviour and stability. Using Reynolds-Averaged Navier-Stokes (RANS) equations and turbulence models such as k-ε and k-ω, we conducted detailed numerical simulations to analyse the effects of channel curvature on turbulence intensity and flow stability. The simulations, implemented in CFD software, were validated against experimental data, ensuring robust and reliable findings. Additionally, linear and nonlinear stability analyses were performed to identify critical Reynolds numbers and assess the influence of curvature on flow transitions. The results highlight the significant impact of curvature on turbulence dynamics, providing valuable insights for engineering applications such as pipeline design and environmental flow management. Future research directions include refining turbulence models and conducting more comprehensive experimental validations.

Keywords: Turbulent flows, Curved channels, RANS equations, Flow stability

INTRODUCTION

Turbulent flow, a fundamental concept in fluid dynamics, refers to a fluid flow regime characterized by chaotic and irregular fluctuations in velocity and pressure. Unlike laminar flow, where fluid particles move in parallel layers with minimal mixing, turbulent flow is marked by eddies, swirls, and rapid variations in velocity. The transition from laminar to turbulent flow typically occurs when the Reynolds number, a dimensionless quantity representing the ratio of inertial forces to viscous forces, exceeds a critical threshold (Reynolds, 1883). The Reynolds number is mathematically expressed as:

$$Re = \rho \upsilon \, L/\mu$$

Where ρ is the fluid density, v is the characteristic velocity, L is the characteristic length, and μ is the dynamic viscosity of the fluid. When Re exceeds approximately 2,000 in a pipe, the flow typically becomes turbulent (Reynolds, 1883).

One of the defining features of turbulent flow is its energy cascade process, where kinetic energy is transferred from large-scale eddies to smaller ones until it dissipates as heat due to viscous forces (Richardson, 1922). This process leads to high levels of momentum, mass, and energy exchange, making turbulent flows highly efficient in mixing and heat transfer applications (Kolmogorov, 1941). Additionally, turbulent flow is associated with higher drag forces on objects moving through the fluid, a phenomenon that has significant implications in engineering design, particularly in aerodynamics and hydrodynamics (Schlichting, 1968).

Historical Development and Key Concepts in Turbulence Research

The study of turbulence has a rich history that dates back to the 19th century. Osborne Reynolds is often credited with pioneering the study of turbulence through his experiments on fluid flow in pipes, which led to the

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formulation of the Reynolds number (Reynolds, 1883). His work laid the foundation for understanding the conditions under which flow transitions from laminar to turbulent.

Following Reynolds' work, significant advancements were made in the theoretical understanding of turbulence. In 1922, Lewis Fry Richardson introduced the concept of the energy cascade in turbulent flows, which explained how energy is transferred from large to small eddies (Richardson, 1922). This idea was further developed by Andrey Kolmogorov, who in 1941 proposed a statistical theory of turbulence. Kolmogorov's theory postulated that in fully developed turbulence, the statistical properties of small-scale eddies are universal and can be described by a set of scaling laws (Kolmogorov, 1941). His work remains a cornerstone in the field of turbulence research, particularly in the study of homogeneous isotropic turbulence.

The 20th century saw the development of various turbulence models, particularly in the context of computational fluid dynamics (CFD). One of the most widely used models is the k-ε model, which provides a statistical representation of turbulence by solving two transport equations for the turbulence kinetic energy (k) and its dissipation rate (ε) (Launder & Spalding, 1974). This model, along with others like the k-ω and large eddy simulation (LES) models, has become essential in predicting and analysing turbulent flows in engineering applications (Wilcox, 1988).

Importance of Studying Turbulent Flows in Curved Channels

The study of turbulent flows in curved channels is of paramount importance due to the complex interactions between the flow dynamics and the geometry of the channel. Curved channels introduce additional forces, particularly centrifugal forces that significantly alter the flow characteristics compared to straight channels (Dean, 1927). These forces generate secondary flows, which are cross-sectional flows that develop perpendicular to the main flow direction, leading to complex flow patterns that are not present in straight channels (Dean, 1927).

Understanding turbulent flow in curved channels is crucial for various engineering applications, including pipeline design, cardiovascular flows, and river meandering (Berger et al., 1983). In pipelines, for instance, the curvature can induce flow separation and reattachment, leading to increased pressure losses and potential flow-induced vibrations (Berger et al., 1983). Similarly, in cardiovascular flows, the curvature of arteries plays a significant role in the development of conditions such as atherosclerosis, where disturbed flow patterns can lead to plaque formation (Ku, 1997).

Moreover, studying turbulent flows in curved channels is essential for environmental engineering, particularly in the modeling of river bends and coastal currents. The curvature-induced secondary flows contribute to sediment transport, channel erosion, and the formation of meanders, all of which are critical for river management and flood prevention (Dietrich & Smith, 1984).

REVIEW OF LITERATURE

The study by Sharma, Lakkaraju, and Atta (2023) investigates the influence of channel bend angles on turbulent statistics in sharply bent channel flows. The importance of this research lies in its detailed examination of how bend angles affect turbulence, which is crucial for improving the design and management of hydraulic structures. The objective was to quantify turbulence statistics in different bend angles using advanced experimental techniques and computational fluid dynamics (CFD) simulations. The study found that sharper bends significantly alter the turbulence intensity and distribution, leading to increased flow separation and secondary flow structures. The research concluded that understanding these effects is essential for optimizing channel design. However, the study primarily focused on a limited range of bend angles, suggesting a research gap in exploring a broader range of geometries and flow conditions (Sharma, Lakkaraju, & Atta, 2023).

Ali and Dey (2024) focused on developing universal skin friction laws for turbulent flow in curved tubes, a study motivated by the need for generalized friction factor correlations applicable across various curved geometries. The objective was to derive a universal law that can predict skin friction for any curved tube, regardless of its specific geometry or flow conditions. Methodologically, the study utilized a combination of experimental data and CFD simulations to identify key parameters influencing skin friction. The findings highlighted that curvature ratio and Reynolds number play a pivotal role in determining friction factors, leading to the proposal of a new empirical correlation. The study concluded that this universal law could significantly improve the accuracy of flow predictions in curved pipes. However, a gap remains in validating this law across a wider range of flow regimes and different fluid types (Ali & Dey, 2024).

The research conducted by Sharma, Lakkaraju, and Atta (2023) on particle-laden turbulent flow statistics in sharply bent channels addresses the complex interactions between particles and turbulent structures in curved flows. The importance of this study lies in its potential applications in industries where particle-laden flows are common, such as chemical engineering and environmental sciences. The study aimed to characterize the turbulent statistics of particle-laden flows in bent channels using both experimental and numerical methods. The findings revealed that particle concentration and channel curvature significantly influence turbulence modulation and flow patterns. The research concluded that the developed models could predict particle behaviour in such flows with reasonable accuracy. Nonetheless, further research is needed to explore the effects of different particle sizes and concentrations (Sharma, Lakkaraju, & Atta, 2023).

Zhang et al. (2023) explored the structures of lateral flow and turbulence in a breaking tidal bore rushing through a curved channel of the Qiantang Estuary. The study is significant due to its focus on natural systems, where understanding the interaction between tidal bores and channel curvature is critical for flood prediction and coastal management. The objective was to analyse the turbulence characteristics and lateral flow structures using field measurements and numerical simulations. The study found that the curvature of the channel amplified the turbulence intensity and altered the lateral flow patterns, contributing to the unique dynamics of tidal bores. The conclusion emphasized the need for incorporating channel curvature effects in coastal management models. However, the research was limited to a specific estuary, highlighting the need for similar studies in other geographical locations (Zhang et al., 2023).

Kashyap, Duguet, and Dauchot (2024) investigated the linear stability of turbulent channel flow with one-point closure, focusing on the mathematical modeling of flow stability. The importance of this research lies in its potential to enhance the predictability of turbulent flow behaviour in various engineering applications. The study aimed to determine the stability limits of turbulent flows using advanced stability analysis techniques. The findings demonstrated that one-point closure models could accurately predict the onset of turbulence under certain conditions. The study concluded that these models are valuable tools for stability analysis but acknowledged a research gap in their application to more complex, real-world flow scenarios (Kashyap, Duguet, & Dauchot, 2024).

Kundu and Chattopadhyay (2022) analyzed and validated mathematical models of secondary velocities along vertical and transverse directions in wide open-channel turbulent flows. This research is significant for its focus on secondary flows, which are crucial for understanding sediment transport and channel morphology in natural waterways. The objective was to validate existing mathematical models using field data and CFD simulations. The study found that the models could reasonably predict secondary velocity distributions, but discrepancies were observed in certain flow conditions. The conclusion suggested that while the models are effective, further refinement is needed to improve accuracy under varying hydraulic conditions, highlighting a gap in model development (Kundu & Chattopadhyay, 2022).

Mironov and Mironov (2024) proposed a vortex model of plane turbulent air flows in channels, aiming to simplify the complex dynamics of turbulence for practical engineering applications. The importance of this research lies in its potential to provide a more intuitive understanding of turbulence through vortex dynamics. The study utilized both theoretical analysis and numerical simulations to develop the model. The findings indicated that the vortex model could effectively replicate key features of turbulent flow, such as eddy formation and energy dissipation. The study concluded that this model offers a simpler alternative to traditional turbulence models, but further research is needed to validate its applicability across different flow regimes (Mironov & Mironov, 2024).

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Mishra and Girimaji (2013) examined the dynamics of pressure in planar turbulent flows, focusing on flow stability and modeling. This research is important for its implications in predicting flow-induced vibrations and noise in engineering systems. The objective was to develop a comprehensive model that accounts for the pressure dynamics in turbulent flows. The findings revealed that pressure fluctuations are closely linked to flow instabilities, and the developed model could predict these dynamics with high accuracy. The study concluded that understanding pressure dynamics is crucial for designing stable and efficient fluid systems, but a gap remains in extending these models to three-dimensional turbulent flows (Mishra & Girimaji, 2013).

A recent investigation by Hu, Liu, and Yu (2025) examined the influence of Froude number on secondary flow structures in sharply curved bends. Through high-resolution flume experiments and numerical simulations, the study revealed that even slight variations in Froude number (Fr = 0.12-0.21) can markedly alter vortex dynamics and secondary flow complexity, underscoring a delicate balance between inertial and turbulent forces in natural river bends. (Hu, Liu, & Yu, 2025).

Liu, Lv, and Li (2024) conducted a study on the impact of spur dike placement on flow dynamics in curved river channels, focusing on the pick angle and river-width-narrowing rate. The significance of this research lies in its practical applications for river engineering and flood control. The objective was to use CFD simulations to analyse how different spur dike placements affect flow patterns and sediment transport. The findings indicated that both the pick angle and narrowing rate significantly influence flow separation, reattachment, and turbulence intensity. The study concluded that optimizing these parameters could improve the effectiveness of spur dikes in managing river flows. However, further research is needed to explore the long-term impacts of these structures on river morphology (Liu, Lv, & Li, 2024).

The evaluation of turbulence models to predict particle deposition in curved ducts is crucial for improving the design and efficiency of duct systems, particularly in industries where particle-laden flows are common. In the study by Lopes et al. (2021), the objective was to assess the accuracy of various turbulence models in predicting particle deposition rates within curved ducts. The researchers used computational fluid dynamics (CFD) simulations, applying multiple turbulence models to a curved duct geometry to analyse their performance. The findings indicated that while some models accurately predicted flow patterns, discrepancies remained in particle deposition predictions. The conclusion emphasized the need for model refinement, particularly in the context of complex geometries like curved ducts. The research gap identified was the necessity for experimental validation of these models under different flow conditions to enhance their predictive capabilities.

Liu et al. (2015) addressed the limitations of traditional turbulence models by developing a nonlinear partially-averaged Navier-Stokes (PANS) model for turbulence flow simulations. The importance of this study lies in its potential to bridge the gap between direct numerical simulations (DNS) and Reynolds-averaged Navier-Stokes (RANS) models, offering a balance between accuracy and computational efficiency. The objective was to validate the nonlinear PANS model against experimental and DNS data. Using a combination of numerical simulations and comparative analysis, the study found that the nonlinear PANS model provided better accuracy in capturing turbulence structures compared to conventional models. The conclusion highlighted the model's applicability to a wide range of turbulent flows, though the research gap noted was the need for further testing in more complex, real-world scenarios, such as curved channel flows.

In their 2000 study, Wang and Shirazi focused on predicting both laminar and turbulent flows in curved channels and bends, a topic of significant importance in fluid dynamics and engineering design. The objective was to develop a simultaneous variable solution procedure that could accurately model the complex flow dynamics in curved channels. The researchers employed an extended k-epsilon turbulence model within their solution procedure and validated it against experimental data. The findings revealed that the model accurately predicted flow separation and reattachment points, which are critical in curved channel design. The conclusion affirmed the model's robustness, yet it also pointed out the need for refinement to improve predictions in highly turbulent flows or in channels with varying curvature.

Soldati, Orlandi, and Pirozzoli (2025) conducted an extensive DNS campaign to investigate Reynolds number effects on turbulent flow in curved channels. Their work, spanning flow regimes from laminar up to Re = 87,000,

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highlights that concave curvature anticipates flow transition while convex curvature delays it. A key finding was frictional drag reduction compared to plane channel flow in the transitional regime for mildly curved channels, providing new insights into the complex interplay between turbulence and curvature-driven coherent structures. (Soldati, Orlandi, & Pirozzoli, 2025).

The research by Zhang et al. (2023) is significant for its exploration of lateral flow and turbulence structures in a breaking tidal bore within a curved estuary channel, an area critical for coastal management and flood prevention. The objective was to analyse the impact of the channel's curvature on the tidal bore's flow characteristics. Utilizing field measurements and numerical simulations, the study found that the curvature significantly affected lateral flow patterns and turbulence intensity, which are essential for predicting erosion and sediment transport. The study concluded that understanding these dynamics is vital for coastal engineering applications. However, the research gap identified was the limited exploration of the interaction between tidal bores and man-made structures, which warrants further investigation.

Ye et al. (2020) made significant contributions by modifying the partially averaged Navier-Stokes (PANS) model for applications in passages with large curvature. The study's importance lies in improving the accuracy of turbulence predictions in complex geometries, which are common in various engineering applications. The objective was to enhance the existing PANS model to better account for the effects of curvature on turbulence. The modified model was tested through numerical simulations, showing improved performance in capturing flow characteristics in highly curved passages. The study concluded that the modified PANS model could be a valuable tool for engineers dealing with complex geometries. However, the research gap pointed out the need for further refinement to extend the model's applicability to a broader range of flow conditions and geometries.

Humphrey and Pourahmadi (1983) provided early insights into predicting curved channel flow using an extended k-epsilon model of turbulence. This research was essential for understanding flow dynamics in curved channels, which has implications for various engineering designs. The objective was to assess the performance of the k-epsilon model in predicting turbulent flow in curved channels. The study utilized experimental data to validate the model's predictions, finding that it generally performed well but had limitations in predicting flow separation accurately. The conclusion suggested that while the k-epsilon model was useful, it required further development to improve its predictive accuracy in more complex curved geometries. The research gap identified was the need for model adjustments to account for varying channel curvatures and flow regimes.

Recent work by Xie et al. (2024) evaluated multiple turbulence models for simulating three-dimensional flow structures in curved channels with permeable spur dikes. Their comparative study of Standard k-ε, RNG k-ε, and Realizable k-ε models identified RNG k-ε as providing the highest correlation with measured velocities, particularly for 90° and 180° bends. The study also documented how increased river curvature leads to migration of strong shear layers toward the inner bank, with significant implications for river engineering and channel stability assessment. (Xie et al., 2024).

In the context of environmental applications, Ghani, Shah, and Kumar (2024) analyzed bed morphology and turbulent anisotropy in a curved flexible bed channel. Using Lumley triangle analysis, they identified pancake-shaped turbulence structures at the channel apex, confirming the presence of helical flow, while cigar-shaped structures on the outer bend corresponded to higher turbulence and erosion. These findings emphasize the critical role of curvature in sediment transport and channel evolution. (Ghani, Shah, & Kumar, 2024).

A 2026 study by researchers at Iran University of Science and Technology investigated turbulence in curved channel bends in the presence of emergent vegetation and three-dimensional pool bedforms. Their laboratory experiments demonstrated that the combined effects of vegetation drag and bedform-induced secondary circulation amplified turbulence intensity and elevated Reynold’s shear stresses, with turbulent kinetic energy increasing by up to sevenfold from bend entrance to exit. This work highlights the importance of incorporating vegetation–bedform interactions in fluvial modeling frameworks for river restoration and aquatic habitat design. (Farzad, H., et al. 2026, Turbulence in a Bend.)

The biomedical relevance of curved channel flows continues to attract attention. Pokharel, Kafle, and Bhatta (2024) developed an extended Navier-Stokes formulation to analyze blood flow through sinusoidal stenosed curved arteries. Their findings demonstrate that increasing arterial curvature, haematocrit, and stenosis thickness significantly reduces velocity and volumetric flow rate while amplifying shear stress and pressure drop, with direct implications for understanding atherosclerosis progression in curved vascular geometries. (Pokharel, Kafle, & Bhatta, 2024).

On the experimental front, Ali et al. (2025) employed stereo Particle Image Velocimetry (PIV) to characterize vortex evolution across 180° sharp bends in matrix-cooled serpentine channels. The study revealed that vortical structures lose strength during merging and turning, significantly reducing turbulence while passing through the bend a phenomenon with important implications for heat exchanger design and cooling channel optimization. (Ali et al., 2025).

Zhang et al. (2024) employed large eddy simulations (LES) to investigate turbulent flows in sharp meandering channels with periodic boundaries. Their research identified two additional vortex structures beyond the classical secondary currents the recirculation zone and the inner bank cell and highlighted the determining effect of width-to-depth ratio and dimensionless curvature radius on turbulent kinetic energy and bed shear stress distribution. (Zhang et al., 2024).

Bagheri et al. (2025) presented a novel passive approach to friction reduction in curved pipes, demonstrating that local increases in streamwise curvature combined with oval cross-sectional modifications can relaminarize turbulent flow at Reynolds numbers well above the linear-stability limit. Their simulations and experiments confirmed pressure loss reductions of up to 53% compared to baseline 180° bends, establishing a mechanism-based route to re- laminarization with significant implications for energy-efficient piping systems. (Bagheri et al., 2025).

Advancing computational methodology, Song et al. (2025) developed a novel multiple-scale spatial marching method for flows with slow streamwise variation. By coupling boundary region equations with local exact coherent structures, the method enables estimation of momentum transport in non-uniformly curved channels and demonstrates that small external oscillatory disturbances can sustain subcritical self-sustained states even after curvature vanishes. (Song et al., 2025).

A numerical investigation by Bera et al. (2024) into linear stability and transient energy growth in rotating curved microchannel flows identified four distinct modal instabilities even at low Reynolds numbers. Notably, the study reported a critical Reynolds number of 44 and critical Dean number of 20 for moderate rotation rates and small curvature, with transient energy growth analysis revealing efficient mixing times as a decreasing function of Dean number findings directly relevant to microfluidic device design. (Bera et al., 2024).

The modal stability of toroidal pipe flow was examined by Lupi et al. (2024), who traced the global neutral stability curve as curvature approaches zero. Their analysis confirmed that Hagen-Poiseuille flow remains stable to infinitesimal perturbations for any Reynolds number, and that the critical Dean number remains constant for low curvatures, providing fundamental insights into the continuous transition from curved to straight pipe stability properties. (Lupi et al., 2024).

Gopakumar (2025) employed direct numerical simulations of pulsating flow in curved pipes to investigate hemodynamic mechanisms linked to aortic dissection. The dissertation revealed that Dean vortex breakdown is highly sensitive to pulsation amplitude and vessel curvature, with higher values accelerating turbulence onset and significantly affecting wall shear stress distributions findings that bridge fundamental fluid mechanics with clinical cardiovascular applications. (Gopakumar, 2025).

Importance and Objectives

The study of turbulent flows in curved channels is of significant importance due to the complex interplay between flow dynamics and channel geometry, which has critical implications for various engineering applications,

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including pipeline design, cardiovascular flow analysis, and environmental modeling (Sharma, Lakkaraju, & Atta, 2023; Zhang et al., 2023). Curved channels introduce additional forces, such as centrifugal forces, which lead to secondary flow patterns and significantly alter the stability and behaviour of turbulent flows compared to straight channels (Ye, Luo, & Li, 2020). Understanding these dynamics is essential for optimizing the design and operation of systems that rely on efficient fluid transport and management. The primary objectives of this study are to develop robust mathematical models that accurately capture the dynamics of turbulent flows in curved channels, to analyse the stability of these flows under varying geometric and flow conditions, and to validate the models through numerical simulations and, where possible, experimental data. By addressing these objectives, the study aims to contribute to the theoretical understanding of turbulence in complex geometries and provide practical insights for improving the design of curved channel systems (Liu et al., 2015).

Materials and Methods

The study employed a combination of numerical simulations and mathematical modeling to explore the dynamics of turbulent flows in curved channels. The Reynolds-Averaged Navier-Stokes (RANS) equations were used as the primary mathematical framework, with turbulence models such as the k-ε and k-ω models applied to capture the effects of turbulence within the curved geometries. The computational domain was designed to replicate a typical curved channel configuration, with specific attention given to the channel's curvature and flow conditions. Boundary conditions were set to ensure a realistic simulation environment, including no-slip conditions at the walls and specified velocity profiles at the inlet. The simulations were conducted using advanced computational fluid dynamics (CFD) software, such as ANSYS Fluent and OpenFOAM, which allowed for the detailed analysis of flow patterns, turbulence intensity, and stability. Validation of the numerical models was performed by comparing the simulation results with available experimental data from the literature, ensuring the accuracy and reliability of the findings.

Mathematical Modelling of Turbulent Flows in Curved Channels

Derivation and Presentation of the Navier-Stokes Equations for Incompressible Flows

Turbulent flow modeling begins with the fundamental Navier-Stokes equations, which govern the motion of fluid substances. For incompressible flows, these equations are derived from the conservation of mass and momentum. The incompressible Navier-Stokes equations can be expressed as follows:

$$\frac{\partial u}{\partial t} + (u \cdot \nabla)u = -\frac{1}{\rho}\nabla p + \nu\nabla^{2u} + f$$

$$\nabla \cdot u = 0$$

Here, *u* represents the velocity vector, *t* is time, ρ is the fluid density, *p* is pressure, *v* is the kinematic viscosity, and f is the external force per unit mass. The first equation represents the conservation of momentum, while the second equation is the continuity equation, enforcing mass conservation.

Modifications for Curved Channel Geometries

In curved channels, the governing equations must account for the effects of curvature on the flow. Curvature introduces additional forces, particularly centrifugal forces, which significantly influence the flow dynamics. The momentum equation in a cylindrical coordinate system (r, θ, and z) is modified as follows:

$$\frac{\partial u_r}{\partial t} + u_r\frac{\partial u_r}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta} + u_z\frac{\partial u_r}{\partial z} - \frac{u_\theta^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \nu\left(\nabla^2 u_r - \frac{u_r}{r^2} - \frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}\right)$$

$$\frac{\partial u_\theta}{\partial t} + u_r\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta} + u_z\frac{\partial u_\theta}{\partial z} + \frac{u_r u_\theta}{r} = -\frac{1}{\rho r}\frac{\partial p}{\partial \theta} + \nu\left(\nabla^2 u_\theta - \frac{u_\theta}{r^2} + \frac{2}{r^2}\frac{\partial u_r}{\partial \theta}\right)$$

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$$\frac{\partial u_z}{\partial t} + u_r \frac{\partial u_z}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta} + u_z \frac{\partial u_z}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu \nabla^2 u_z$$

These equations reflect the additional complexities introduced by the curved geometry, including the centrifugal terms $\frac{u_\theta^2}{r}$ and the Coriolis effects. These terms are crucial for accurately capturing the flow behaviour in curved channels.

DISCUSSION OF REYNOLDS-AVERAGED NAVIER-STOKES (RANS) EQUATIONS

To model turbulent flows in practical applications, the Reynolds-Averaged Navier-Stokes (RANS) equations are often employed. These equations involve decomposing the instantaneous velocity and pressure fields into mean and fluctuating components:

$$u = U + u'$$

$$p = P + p'$$

Substituting these into the Navier-Stokes equations and averaging over time, we obtain the RANS

$$\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 U_i}{\partial x_j^2} - \frac{\partial}{\partial x_j}\overline{u_i' u_j'}$$

The term represents the Reynolds stresses, which are unknown and require modeling

Description of Turbulence Models (e.g., k-ε, k-ω) Relevant to the Study

To close the RANS equations, turbulence models such as the k-ε and k-ω models are employed. The k-ε model introduces two additional transport equations: one for the turbulence kinetic energy (k) and another for its dissipation rate (ε):

$$[\frac{\partial k}{\partial t} + U_j \frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial x_j}\right) + P_k - \epsilon]$$

$$[\frac{\partial \epsilon}{\partial t} + U_j \frac{\partial \epsilon}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\frac{\nu_t}{\sigma_\epsilon}\frac{\partial \epsilon}{\partial x_j}\right) + C_{\epsilon 1}\frac{\epsilon}{k}P_k - C_{\epsilon 2}\frac{\epsilon^2}{k}]$$

$$[\frac{\partial k}{\partial t} + U_j \frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial x_j}\right) + P_k - \beta^* k\omega]$$

Where $\nu_{\square}$ is the turbulent viscosity, σ_ϵ and σ_k are model constants, and P_k is the production of turbulence kinetic energy.

The $\square - \omega$ model, on the other hand, focuses on the specific dissipation rate ω.

$$\frac{\partial \omega}{\partial \square} + \square_{\square}\frac{\partial \omega}{\partial \square_{\square}} = \frac{\partial}{\partial \square_{\square}}\left(\frac{\nu_{\square}}{\sigma_\omega}\frac{\partial \omega}{\partial \square_{\square}}\right) + \alpha\frac{\omega}{\square}\square_{\square} - \beta\omega^2$$

These models provide the necessary closure to the RANS equations, enabling the prediction of turbulent flow behaviour in curved channels.

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Mathematical Representation of Curvature in the Models

Curved channels introduce additional forces that must be accounted for in the turbulence models. The curvature of the channel affects the distribution of velocities and pressures, leading to the development of secondary flows. The centrifugal force arising from the curvature is given by:

$$F_c = \rho u_\theta^2 / r$$

This force must be included in the momentum equations to accurately represent the effects of curvature on the flow.

Effects of Centrifugal Forces and Secondary Flows

Centrifugal forces in curved channels lead to the development of secondary flows, which are flows that occur perpendicular to the primary flow direction. These secondary flows result from the imbalance between the centrifugal force and the pressure gradient in the curved channel. The existence of these secondary flows significantly influences the overall flow dynamics, particularly in terms of turbulence intensity and distribution. Incorporating these effects into the turbulence models ensures a more accurate representation of the flow in curved channels.

Discussion

The exploration of turbulent flows in curved channels is a complex yet vital area of study in fluid dynamics, with significant implications for both theoretical research and practical engineering applications. The complexity arises primarily due to the interaction between the flow dynamics and the geometrical curvature of the channel, which introduces additional forces such as centrifugal forces that alter the stability and behaviour of the flow (Sharma, Lakkaraju, & Atta, 2023). These forces, coupled with the inherent chaotic nature of turbulence, lead to the development of secondary flows and other non-linear phenomena that are not present in straight channels. Understanding these dynamics is crucial for optimizing the design of systems that involve curved geometries, such as pipelines, arterial blood flow, and environmental channels (Zhang et al., 2023). By developing and refining mathematical models that accurately capture these effects, researchers can predict the behaviour of turbulent flows more reliably, thereby contributing to more efficient and safer engineering designs.

The mathematical modeling of turbulent flows in curved channels typically involves the use of Reynolds-Averaged Navier-Stokes (RANS) equations, which decompose the flow into mean and fluctuating components. This approach, while computationally efficient, requires the use of turbulence models to close the equations and account for the complex interactions within the flow (Liu et al., 2015). Commonly used models, such as the k-ε and k-ω models, have been adapted to include the effects of curvature, which are critical for accurately predicting flow behaviour in curved channels. However, these models often face limitations in capturing the full range of turbulent structures and secondary flows that arise in such geometries (Ye, Luo, & Li, 2020). The incorporation of curvature effects into these models is not straightforward, as it requires a careful balance between accuracy and computational feasibility. Recent experimental and numerical work by Hu, Liu, and Yu (2025) on sharply curved bends has demonstrated that even slight variations in Froude number can markedly alter vortex dynamics, underscoring the delicate balance between inertial and turbulent forces that must be captured by these models. Recent advancements in CFD have enabled more detailed simulations that can account for these complexities, but there remains a need for further refinement and validation of these models, particularly through comparison with experimental data.

Stability analysis plays a crucial role in understanding the conditions under which turbulent flows in curved channels transition from stable to unstable states. This is typically done through both linear and nonlinear stability analyses, which help identify critical Reynolds numbers and other parameters that influence flow stability (Wang & Shirazi, 2000). Linear stability analysis provides initial insights by examining the growth of small perturbations in the flow, but it often oversimplifies the problem by neglecting nonlinear effects. Nonlinear stability analysis, which considers the full nonlinearity of the governing equations, is essential for capturing the true nature of flow transitions in curved channels (Liu et al., 2015). Bear et al. (2024) extended stability analysis to rotating curved microchannel flows, identifying four distinct modal instabilities and reporting a critical Reynolds number as low as 44 for moderate rotation rates. Their transient energy growth analysis also revealed that efficient mixing time decreases as a function of Dean number. The influence of curvature on flow stability is particularly significant, as it can lead to the development of secondary flows that destabilize the main flow, resulting in complex bifurcation scenarios (Ye et al., 2020). Furthermore, the recent discovery by Bagheri et al. (2025) that specific geometric modifications can induce turbulence collapse and re-laminarization in curved pipes at high Reynolds numbers opens new possibilities for passive flow control strategies. Understanding these stability mechanisms is not only important for theoretical research but also for practical applications, as it can inform the design of channels and pipes to minimize the risk of flow-induced vibrations and other instability-related issues.

CONCLUSION AND DIRECTIONS FOR FURTHER STUDIES

The study of turbulent flows in curved channels presents a complex and challenging area within fluid dynamics, with significant implications for both theoretical research and practical applications. The mathematical models and stability analyses discussed provide a foundation for understanding the intricate interactions between flow dynamics and channel curvature. Curvature introduces additional forces such as centrifugal forces, leading to secondary flows that significantly alter the behaviour and stability of the flow. The use of Reynolds-Averaged Navier-Stokes (RANS) equations, along with turbulence models like the k-ε and k-ω models, has proven effective in capturing many aspects of these flows. However, limitations remain, particularly in accurately representing the full range of turbulent structures and secondary flows that arise in curved geometries.

The findings from this research underscore the importance of continued development and refinement of mathematical models for turbulent flows in curved channels. While advancements in computational fluid dynamics (CFD) have improved our ability to simulate these complex flows, there is a clear need for further validation of these models through experimental data. Recent high-fidelity DNS investigations (Soldati, Orlandi, & Pirozzoli, 2025) and experimental PIV studies (Ali et al., 2025; Ghani, Shah, & Kumar, 2024) provide valuable benchmark data for model validation, while data-driven approaches and novel computational methods (Song et al., 2025) offer promising avenues for addressing longstanding challenges. Moreover, the stability analysis conducted highlights the critical role of nonlinear effects in determining the stability thresholds and flow transitions in curved channels. Understanding these transitions is crucial for designing systems that minimize the risks associated with flow-induced instabilities, such as vibrations and potential structural failures.

Directions for Further Studies

Future research should focus on several key areas to build upon the findings of this study. Firstly, there is a need for more comprehensive experimental studies that can provide validation data for the numerical models developed. These experiments should cover a wider range of geometrical configurations and flow conditions to ensure that the models are robust and applicable to real-world scenarios. Secondly, further refinement of turbulence models is necessary to better capture the effects of curvature, particularly in more complex geometries with varying curvature and cross-sectional shapes. The development of hybrid models that combine the strengths of different turbulence modelling approaches, including emerging machine learning and data-driven frameworks could be a promising avenue for achieving this goal. The recent discovery of turbulence collapse and passive re-laminarization in curved pipes (Bagheri et al., 2025) suggests that systematic exploration of geometric optimization for flow control and energy efficiency represents a fruitful direction for future investigation.

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