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Nonlinear Rational Displacement Contractions in G-Metric Spaces: Fixed Points, Best Proximity Points, Coupled Fixed Points, and Applications to Fractional Differential Equations

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ABSTRACT

In this paper, a nonlinear rational displacement contraction is introduced in the setting of G-metric spaces. The proposed contractive condition combines the classical three-point G-distance with a nonlinear rational interaction involving the displacement of points under a self-mapping. Unlike several hybrid contractive formulations in which the convergence factor is imposed separately, the present framework derives the effective Picard contraction constant directly from the parameters of the proposed inequality. Under the condition $\lambda + \eta < 1$, it is shown that the Picard iteration satisfies a geometric estimate with contraction factor $q = \frac{\lambda}{1-\eta} < 1$. Consequently, existence and uniqueness of a fixed point are established in a G-complete space, together with convergence and an explicit error estimate for the iterative sequence.The framework is further extended to best proximity points for non-self mappings and to coupled fixed points through an induced mapping on a product space. Finally, an application to a nonlinear Caputo fractional differential equation is presented through an integral operator formulation that actively incorporates the rational displacement structure. The results provide a nonlinear rational alternative for generalized contractive mappings in G-metric spaces and establish a connection between abstract fixed point theory and fractional differential equations.

Keywords:G-metric space; rational contraction; nonlinear contraction; displacement mapping; fixed point; best proximity point; coupled fixed point; fractional differential equation.

MSC (2020): 47H10; 54H25; 34A08; 34A12.

INTRODUCTION

Fixed point theory is an important branch of nonlinear analysis and has numerous applications in differential equations, integral equations, optimization, dynamic systems, and nonlinear operator theory [1] The Banach contraction principle remains one of the most fundamental results in this area and has motivated the development of a large number of generalized contractive conditions [2].Generalized metric structures provide an important direction for extending classical fixed-point theory. Among these structures, the notion of a G-metric space introduced by Mustafa and Sims [3-4] replaces the ordinary two-point distance by a function defined on triples of points. This framework makes it possible to formulate contractive conditions involving three-point interactions and has generated a substantial body of fixed point results. In recent years, fixed point principles in fuzzy metric domains have evolved substantially through the investigation of generalized contractive constraints and implicit functional inequalities. In particular, Mishra et al. [9] proved common fixed point theorems in M-fuzzy metric spaces governed by integral-type implicit relations. The convergence dynamics of fuzzy contractive mappings under integral inequalities were further established in [10]. Concurrently, constructive computational techniques and applications of fuzzy fixed point iterations were developed in [11], whereas the properties of weakly compatible operators under integral-type contractive inequalities were analyzed in [12].

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Another important direction is the introduction of nonlinear contractive conditions. Such conditions may involve displacement terms, rational expressions, maximum-type functionals, simulation functions, or other nonlinear control mechanisms. A principal challenge in constructing a useful generalized contraction is to ensure that the parameter assumptions directly produce a convergent iterative process.

The purpose of this paper is to introduce a nonlinear rational displacement contraction in a G-metric space. The proposed condition is given

$$G(Tx, Ty, Tz) \leq \lambda G(x, y, z) + \eta \frac{G(x,Tx,Tx)G(y,Ty,Ty)}{1+G(x,y,z)}, \tag{1}$$

where $\lambda, \eta \geq 0$ and

$$\lambda + \eta < 1. \tag{2}$$

The central feature of the proposed framework is that the effective contraction constant for the Picard iteration is not introduced as an independent assumption. Instead, it follows directly from the contractive parameters:

$$q = \frac{\lambda}{1-\eta} < 1.$$

The main objectives of this paper are:

1. to introduce a nonlinear rational displacement contraction in G-metric spaces;
2. to establish existence and uniqueness of fixed points in G-complete spaces;
3. to derive convergence and an explicit error estimate for Picard iteration;
4. to formulate a best proximity point principle for non-self mappings;
5. to obtain a coupled fixed point result through a product-space construction;
6. to demonstrate an application to a nonlinear Caputo fractional differential equation.

Preliminaries

We recall some basic definitions and properties of G-metric spaces from Mustafa and Sims [3].

Definition 1. Let X be a nonempty set. A function $G: X \times X \times X \to [0, \infty)$ is called a G-metric if, for all $x, y, z, a \in X$, the following conditions are satisfied:

1. $G(x, y, z) = 0 \Leftrightarrow x = y = z$;
2. $G(x, x, y) > 0$ whenever $x \neq y$;
3. $G(x, x, y) \leq G(x, y, z)$ whenever $y \neq z$;
4. $G(x, y, z) = G(x, z, y) = G(y, x, z) =$ (symmetry in all three variables);
5. $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$ (rectangle inequality).

The pair (X, G) is called a G-metric space.

Definition 2. A sequence $\{x_n\} \subset X$ is said to G-converge to $x \in X$ if

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$$\lim_{n,m\to\infty} G(x, x_n, x_m) = 0.$$

Definition 3. A sequence $\{x_n\} \subset X$ is called G-Cauchy if for every $\epsilon > 0$, there exists $N \in \mathbb{N}$ such that $G(x_n, x_m, x_l) < \epsilon$ for all $n, m, l \geq N$.

Definition 4. A G-metric space (X, G) is called G-complete if every G-Cauchy sequence in X is G-convergent.

Lemma 5. For any $x, y, z \in X$, $G(x, y, z) \leq G(x, x, y) + G(x, x, z)$. (4)

Proof. The result follows directly from the rectangle inequality (G5) and the symmetry property (G4) of Definition 1.

Nonlinear Rational Displacement Contractions

We now formulate the primary contractive notion investigated in this work.

Definition 6 (Nonlinear Rational Displacement G-Contraction). Let (X, G) be a G-metric space and let $T: X \to X$ be a self-mapping. The mapping T is called a nonlinear rational displacement G-contraction if there exist constants $\lambda, \eta \geq 0$ satisfying

$$\lambda + \eta < 1 \tag{5}$$

such that

$$G(Tx, Ty, Tz) \leq \lambda G(x, y, z) + \eta \frac{G(x, Tx, Tx) G(y, Ty, Ty)}{1 + G(x, y, z)} \tag{6}$$

for all $x, y, z \in X$.

Remark 7. The nonlinear rational term in (6) involves the product of displacements under T. The constraint (5) ensures geometric convergence of the Picard iteration without requiring any ad-hoc Lipschitz assumptions.

The Fundamental Iterative Estimate

Lemma 8 (Iterative Estimate). Let (X, G) be a G-metric space and let $T: X \to X$ satisfy condition (6)of Definition 6. Let $x_0 \in X$ and define the Picard sequence by

$$x_{n+1} = Tx_n, \quad n \geq 0. \tag{7}$$

Then for all $n \geq 1$,

$$G(x_n, x_{n+1}, x_{n+1}) \leq q G(x_{n-1}, x_n, x_n) \tag{8}$$

Where

$$q = \frac{\lambda}{1-\eta} < 1. \ (9)$$

Proof. Define $d_n = G(x_n, x_{n+1}, x_{n+1})$ for each $n \geq 0$. Setting $x = x_{n-1}$ and $y = z = x_n$ in (6). we obtain

$$\begin{aligned} d_n &= G(Tx_{n-1}, Tx_n, Tx_n) \\ &\leq \lambda G(x_{n-1}, x_n, x_n) + \eta \frac{G(x_{n-1}, Tx_{n-1}, Tx_{n-1}) G(x_n, Tx_n, Tx_n)}{1 + G(x_{n-1}, x_n, x_n)}. \end{aligned}$$

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Since $Tx_{n-1} = x_n$ and $Tx_n = x_{n+1}$ by (6), this inequality becomes

$$d_n \le \lambda d_{n-1} + \eta \frac{d_{n-1} d_n}{1 + d_{n-1}}.$$

Using the elementary bound $\frac{d_{n-1}}{1+d_{n-1}} \le 1$, we have

$$d_n \le \lambda d_{n-1} + \eta d_n \Longrightarrow (1-\eta) d_n \le \lambda d_{n-1}.$$

Since $\lambda + \eta < 1$, it follows that $1 - \eta > \lambda \ge 0$, which yields

$$d_n \le \frac{\lambda}{1-\eta} d_{n-1} = q d_{n-1},$$

with $0 \le q < 1$, establishing (8).

Main Fixed-Point Result

Theorem 9 (Existence and Uniqueness). Let (X, G) be a G-complete G-metric space and let $T: X \to X$ be a nonlinear rational displacement G-contraction satisfying (5) and (6). Then T possesses a unique fixed point $u^* \in X$. Moreover, for every initial point $x_0 \in X$, the Picard sequence $\{x_n\}$ defined by (7)G-converges to u^*.

Proof. Let $x_0 \in X$ be arbitrary and consider the Picard sequence $\{x_n\}$ from (7). By Lemma 8, we have by induction

$$d_n \le q^n d_0, \quad n \ge 0. \tag{10}$$

For any $m, n \in \mathbb{N}$ with $m > n$, repeated application of the rectangle inequality (G5) yields

$$G(x_n, x_m, x_m) \le \textstyle\sum_{k=n}^{m-1} G(x_k, x_{k+1}, x_{k+1}) = \sum_{k=n}^{m-1} d_k. \tag{11}$$

Applying (10) to (11), we obtain

$$G(x_n, x_m, x_m) \le d_0 \sum_{k=n}^{m-1} q^k \le \frac{q^n}{1-q} d_0.$$

Because $q \in [0,1)$, $\lim_{n\to\infty} \frac{q^n}{1-q} d_0 = 0$, which implies that $\{x_n\}$ is a G-Cauchy sequence according to (Definition 3). By G-completeness (Definition 4), there exists $u^* \in X$ such that

$$\lim_{n\to\infty} x_n = u^*. \tag{12}$$

Next, we establish that $Tu^* = u^*$. Applying condition (6) with $x = x_n$ and $y = z = u^*$,

$$G(x_{n+1}, Tu^*, Tu^*) = G(Tx_n, Tu^*, Tu^*) \le \lambda G(x_n, u^*, u^*) + \eta \frac{G(x_n, x_{n+1}, x_{n+1}) G(u^*, Tu^*, Tu^*)}{1 + G(x_n, u^*, u^*)}.$$

Passing to the limit as $n \to \infty$ and utilizing (12) and $d_n \to 0$, we obtain

$$G(u^*, Tu^*, Tu^*) \le 0 \Longrightarrow G(u^*, Tu^*, Tu^*) = 0,$$

which implies $Tu^* = u^*$ by property (G1).

To show uniqueness, suppose $v^* \in X$ is another fixed point of T. Setting $x = u^*$ and $y = z = v^*$ in (6)

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$$G(u^*, v^*, v^*) = G(Tu^*, Tv^*, Tv^*) \leq \lambda G(u^*, v^*, v^*) + \eta \frac{G(u^*, Tu^*, Tu^*)G(v^*, Tv^*, Tv^*)}{1 + G(u^*, v^*, v^*)}.$$

Since $Tu^* = u^*$ and $Tv^* = v^*$, the numerator of the rational term vanishes identically, giving

$$G(u^*, v^*, v^*) \leq \lambda G(u^*, v^*, v^*).$$

Since $\lambda < 1 - \eta \leq 1$, this inequality requires $G(u^*, v^*, v^*) = 0$, which implies $u^* = v^*$ by (G1).

Corollary 10 (A Priori Error Estimate). Under the assumptions of Theorem 9, the Picard sequence satisfies the error bound $G(x_n, u^*, u^*) \leq \frac{q^n}{1-q} G(x_0, x_1, x_1)$, (13)

where q is defined in (9).

Proof. Letting $m \to \infty$ in (11) and utilizing the convergence $x_m \to u^*$ from Theorem 9 yields (13). □

Illustrative Example

Example 11. Let $X = [0,1]$ equipped with the standard G-metric

$$G(x, y, z) = |x - y| + |y - z| + |z - x|.$$

The space (X, G) is G-complete. Define the mapping $T: X \to X$ by

$$T(x) = \frac{x^2}{6}.$$

For any $x, y, z \in X$, assume without loss of generality that $x \geq y \geq z$. Then

$$G(Tx, Ty, Tz) = 2|Tx - Tz| = \frac{x^2 - z^2}{3} = \frac{x + z}{3}(x - z).$$

Since $x, z \in [0,1]$, we have $\frac{x+z}{3} \leq \frac{2}{3}$, which yields

$$G(Tx, Ty, Tz) \leq \frac{1}{3} \cdot 2(x - z) = \frac{1}{3} G(x, y, z).$$

Choose parameters

$$\lambda = \frac{1}{3}, \quad \eta = \frac{1}{4} > 0.$$

Then $\lambda + \eta = \frac{1}{3} + \frac{1}{4} = \frac{7}{12} < 1$, satisfying (5). Because the rational displacement term in (6) is strictly non-negative,

$$G(Tx, Ty, Tz) \leq \frac{1}{3} G(x, y, z) \leq \lambda G(x, y, z) + \eta \frac{G(x, Tx, Tx)G(y, Ty, Ty)}{1 + G(x, y, z)}$$

holds for all $x, y, z \in X$ with $\eta > 0$. The effective Picard contraction constant is computed via (9) as

$$q = \frac{\lambda}{1 - \eta} = \frac{1/3}{1 - 1/4} = \frac{4}{9} < 1.$$

Hence, Theorem 9 applies directly, and $u^* = 0$ is verified as the unique fixed point in [0,1].

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Best Proximity Points

We extend the framework to non-self mappings following proximal techniques in generalized spaces [5]. Let A and B be nonempty subsets of a G-metric space (X, G).

Definition 12. Define the distance between sets and the corresponding boundary subsets by:

$$G(A,B,B) = \inf\{G(x,y,y): x \in A, y \in B\},$$
$$A_0 = \{x \in A: G(x,y,y) = G(A,B,B) \text{ for some } y \in B\},$$
$$B_0 = \{y \in B: G(x,y,y) = G(A,B,B) \text{ for some } x \in A\}.$$

Definition 13. Let $T: A \to B$. A point $x^* \in A$ is called a best proximity point of T if

$$G(x^*, Tx^*, Tx^*) = G(A,B,B).$$

Definition 14. The pair (A, B) is said to satisfy the G-P property if

$$\left.\begin{matrix} G(x_1,y_1,y_1) = G(A,B,B) \\ G(x_2,y_2,y_2) = G(A,B,B) \end{matrix}\right\} \Longrightarrow G(x_1,x_2,x_2) = G(y_1,y_2,y_2).$$

Definition 15. A mapping $T: A \to B$ is called a rational displacement proximal G-contraction if there exist $\lambda, \eta \geq 0$ satisfying $\lambda + \eta < 1$ such that whenever

$$G(u,Tx,Tx) = G(A,B,B) \quad \text{and} \quad G(v,Ty,Ty) = G(A,B,B), \tag{16}$$

one has

$$G(u,v,v) \leq \lambda G(x,y,y) + \eta \frac{G(x,u,u)G(y,v,v)}{1+G(x,y,y)}. \tag{17}$$

Theorem 16. Let (X, G) be a G-complete G-metric space and let A, B be nonempty closed subsets of X. Assume that:

1. $A_0 \neq \emptyset$ and A_0 is closed;
2. $T(A_0) \subseteq B_0$;
3. The pair (A, B) satisfies the G-P property;
4. $T: A \to B$ is a rational displacement proximal G-contraction satisfying (17).

Then T possesses a unique best proximity point in A_0.

Proof. Let $x_0 \in A_0$. Since $T(A_0) \subseteq B_0$, there exists $x_1 \in A_0$ such that $G(x_1, Tx_0, Tx_0) = G(A,B,B)$. Inductively, construct a sequence $\{x_n\} \subset A_0$ satisfying

$$G(x_{n+1}, Tx_n, Tx_n) = G(A,B,B) \quad \text{for all } n \geq 0. \tag{18}$$

Applying (17) to (18) with $u = x_n, v = x_{n+1}, x = x_{n-1}, y = x_n$, we deduce

$$G(x_n, x_{n+1}, x_{n+1}) \leq \lambda G(x_{n-1}, x_n, x_n) + \eta \frac{G(x_{n-1}, x_n, x_n)G(x_n, x_{n+1}, x_{n+1})}{1 + G(x_{n-1}, x_n, x_n)}.$$

Following the derivation of Lemma 8, we obtain

$$G(x_n, x_{n+1}, x_{n+1}) \leq qG(x_{n-1}, x_n, x_n),$$

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where $q = \frac{\lambda}{1-\eta} < 1$. Consequently, $\{x_n\}$ is a G-Cauchy sequence in A_0. Since A_0 is closed in the G-complete space (X, G), there exists $x^* \in A_0$ such that $x_n \to x^*$. Taking the limit in (18) gives $G(x^*, Tx^*, Tx^*) = G(A, B, B)$, proving that x^* is a best proximity point. Uniqueness follows directly as in Theorem 9.

Coupled Fixed Points

Let $F: X \times X \to X$ be a given operator [6].

Definition 17. An element $(u, v) \in X \times X$ is called a coupled fixed point of F if

$$F(u, v) = u \quad \text{and} \quad F(v, u) = v. \tag{19}$$

Define the mapping $\mathcal{F}: X \times X \to X \times X$ by

$$\mathcal{F}(x, y) = (F(x, y), F(y, x)). \tag{20}$$

For elements $A = (x_1, y_1), B = (x_2, y_2), C = (x_3, y_3) \in X \times X$, define the product G-metric

$$G_\times(A, B, C) = G(x_1, x_2, x_3) + G(y_1, y_2, y_3). \tag{21}$$

Definition 18 (Coupled Rational Displacement Contraction). A mapping $F: X \times X \to X$ is called a coupled rational displacement contraction if there exist constants $\lambda, \eta \geq 0$ with $\lambda + \eta < 1$ such that

$$G_\times(\mathcal{F}A, \mathcal{F}B, \mathcal{F}C) \leq \lambda G_\times(A, B, C) + \eta \frac{G_\times(A, \mathcal{F}A, \mathcal{F}A) G_\times(B, \mathcal{F}B, \mathcal{F}B)}{1 + G_\times(A, B, C)}$$

for all $A, B, C \in X \times X$, where $\mathcal{F}$ and $G_\times$ are defined by (20) and (21).

Theorem 19 (Coupled Fixed Point Theorem). Let (X, G) be a G-complete G-metric space. Suppose that $F: X \times X \to X$ satisfies Definition 18 with condition [eq:coupled_contraction]. Then F has a unique coupled fixed point in $X \times X$.

Proof. Equip the Cartesian product $X \times X$ with the metric $G_\times$ defined in (21). It is well known that $(X \times X, G_\times)$ is G-complete whenever (X, G) is G-complete. In view of (22), the induced operator $\mathcal{F}: X \times X \to X \times X$ satisfies the nonlinear rational displacement contraction (6) of Definition 6 on $(X \times X, G_\times)$.

By Theorem 9, $\mathcal{F}$ possesses a unique fixed point $A^* = (u, v) \in X \times X$. Hence,

$$(u, v) = \mathcal{F}(u, v) = (F(u, v), F(v, u)),$$

which implies $F(u, v) = u$ and $F(v, u) = v$. Uniqueness of A^* in $X \times X$ ensures the uniqueness of the coupled fixed point.

Application to a Fractional Differential Equation

Fractional differential equations are central to modeling anomalous diffusion and memory phenomena [7,8]. Consider the nonlinear Caputo fractional differential boundary value problem:

$$^{c}\mathcal{D}^{\delta} u(t) = f(t, u(t)), \quad t \in [0,1], \quad 1 < \delta \leq 2, \tag{23}$$

subject to boundary constraints whose Green function inversion gives the equivalent integral equation:

$$u(t) = \int_0^1 H(t, s) f(s, u(s))\, ds, \tag{24}$$

where $H(t, s)$ is the associated continuous Green function [7].

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Let $X = C([0,1], \mathbb{R})$ be endowed with the supremum norm $\| u \|_\infty = \max_{t\in[0,1]} |u(t)|$ and the standard induced G-metric:

$$G(u, v, w) = \| u - v \|_\infty + \| v - w \|_\infty + \| w - u \|_\infty. \tag{25}$$

The space (X, G) is complete. Let

$$K = \sup_{t\in[0,1]} \int_0^1 | H(t, s)| \, ds. \tag{26}$$

To engage the full nonlinear framework where $\eta > 0$, consider the perturbed integral operator $T: X \to X$ defined by

$$(Tu)(t) = \int_0^1 H\,(t, s) f(s, u(s))\, ds + \alpha \frac{\|u - Tu\|_\infty}{1 + \|u\|_\infty} \psi(t), \tag{27}$$

where $\psi \in C([0,1], [0,1])$ with $\| \psi \|_\infty \leq 1$, and $\alpha \geq 0$ is a coupling coefficient.

Theorem 20. Suppose that:

1. There exists $L > 0$ such that $|f(t, x) - f(t, y)| \leq L|x - y|$ for all $t \in [0,1]$ and $x, y \in \mathbb{R}$;
2. The parameters satisfy $\lambda = 2LK$ and $\eta = \frac{\alpha}{2} > 0$ such that $\lambda + \eta < 1$.

Then the fractional operator T defined by (27) possesses a unique continuous solution $u^* \in C([0,1], \mathbb{R})$.

Proof. For any $u, v \in X$ and $t \in [0,1]$, equation (27) and condition (i) imply

$$\begin{aligned} |(Tu)(t) - (Tv)(t)| &\leq \int_0^1 | H(t, s)| \, |f(s, u(s)) - f(s, v(s))| \, ds + \alpha \frac{\| u - Tu \|_\infty}{1 + \| u \|_\infty} |\psi(t)| \\ &\leq L \| u - v \|_\infty \int_0^1 | H(t, s)| \, ds + \alpha \frac{\| u - Tu \|_\infty \| v - Tv \|_\infty}{1 + \| u - v \|_\infty}. \end{aligned}$$

Taking the supremum over $t \in [0,1]$ and using (26),

$$\| Tu - Tv \|_\infty \leq LK \| u - v \|_\infty + \alpha \frac{\|u - Tu\|_\infty \|v - Tv\|_\infty}{1 + \|u - v\|_\infty}. \tag{28}$$

Using (25), we compute $G(Tu, Tv, Tw) = \| Tu - Tv \|_\infty + \| Tv - Tw \|_\infty + \| Tw - Tu \|_\infty$. Summing the bounds from (28) over all cyclic permutations yields

$$G(Tu, Tv, Tw) \leq \lambda G(u, v, w) + \eta \frac{G(u, Tu, Tu) G(v, Tv, Tv)}{1 + G(u, v, w)},$$

with $\lambda = 2LK$ and $\eta = \frac{\alpha}{2} > 0$. Since $\lambda + \eta < 1$, Theorem 9 applies directly, proving that T possesses a unique fixed point $u^* \in C([0,1], \mathbb{R})$.

DISCUSSION

The principal feature of the proposed framework is the nonlinear rational term

$$\frac{G(x, Tx, Tx) G(y, Ty, Ty)}{1 + G(x, y, z)}.$$

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This term incorporates point displacement and produces a convergent Picard iteration with effective contraction constant

$$q = \frac{\lambda}{1-\eta} < 1.$$

CONCLUSION

A nonlinear rational displacement contraction has been introduced in the setting of G-metric spaces. The principal result establishes existence and uniqueness of fixed points in a G-complete space under the parameter condition $\lambda + \eta < 1$. The fundamental iterative estimate produces the explicit contraction factor

$$q = \frac{\lambda}{1-\eta} < 1,$$

which guarantees the convergence of the Picard iteration. An explicit a priori error estimate has also been obtained, along with extensions to best proximity points and coupled fixed points, and an application to nonlinear fractional differential equations.

REFERENCES

1. R. P. Agarwal, M. Meehan, and D. O'Regan, Fixed Point Theory and Applications, Cambridge University Press, Cambridge, 2001.
2. S. K. Chatterjea, Fixed-point theorems, C. R. Acad. Bulgare Sci., 25 (1972), 727–730.
3. Z. Mustafa and B. Sims, A new approach to generalized metric spaces, J. Nonlinear Convex Anal., 7(2) (2006), 289–297. Z.
4. Mustafa, A new structure for generalized metric spaces with applications to fixed point theory, Int. J. Math. Math. Sci., 2006 (2006), Article ID 90306.
5. S. S. Basha, Best proximity points: Global optimal solutions, J. Global Optim., 49(1) (2011), 15–21.
6. T. G. Bhaskar and V. Lakshmikantham, Fixed point theorems in partially ordered metric spaces and applications, Nonlinear Anal., 65(7) (2006), 1379–1393.

A. Kilbas, H. M. Srivastava, and J. J. Trujillo, Theory and Applications of Fractional Differential Equations, Elsevier, Amsterdam, 2006.

7. Podlubny, Fractional Differential Equations, Academic Press, San Diego, 1999.
8. M. K. Mishra et al., A Common Fixed Point Theorem in M-fuzzy Metric Spaces Satisfying Integral Type Implicit Relations, Res. J. Appl. Sci. Eng. Technol. 2 (2010), no. 5, 418–421.
9. M. K. Mishra et al., On Common Fixed Point Theorems in Fuzzy Metric Spaces Satisfying Integral Type Inequality, Res. J. Appl. Sci. Eng. Technol. 2 (2010), no. 8, 727–733.
10. M. K. Mishra et al., An application of Fixed Point Theorems in Fuzzy Metric Spaces, Int. J. Adv. Eng. Sci. Technol. 11 (2010), no. 2, 123–129.
11. M. K. Mishra et al., Fixed Points theorem in Fuzzy Metric Space for weakly Compatible Maps satisfying Integral type Inequality, Int. J. Appl. Eng. Res. Dindigul 2 (2010), no. 1, 28–36.