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A Comparison of the Vasicek and Cox, Ingersoll, and Ross Interest Rate Models in Valuation of Insurance Assets, Liabilities and Surplus

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Abstract: Insurance Company's cash flows are subjected to the risk of interest rate (C-3 risk). To curb the effect of this risk, Insurance companies normally adopts an interest period model that predicts the movement of the rates of interest. The most common models adopted by the Insurance Companies are the vasicek (1977) model and The Cox, Ingersoll and Ross (1985) Model. These two models are stochastic single period short-rate models; however, they exhibit different assumptions and because of this, the future values of insurance Assets and liabilities are likely to differ when these models are applied to estimate their values. Valuing of Insurance Assets and liabilities, especially in the Kenyan market is very challenging because of the tremendous fluctuations of interest rates as a result of gradual increments of the rate of inflation. In order for insurance companies to correctly value their insurance policies, they need to have a substantive Knowledge of their cash flows. The current valuation methods of insurance assets, liabilities and Surplus based on a stochastic interest rate models do not consider the possibility of occurrence of model risk, and therefore there is a possibility of either under estimating the future values of insurance assets and liabilities or over estimating. In this research paper, Geometric simulation was used to explore the effect of model risk By creating a comparison between The vacisek and the Cox, Ingersoll and Ross interest rate model. First, we evaluated the value of an insurance company's assets and liabilities by assuming that the interest rate process is followed by the Cox, Ingersoll and Ross model and The vasicek (1977). Model risk arose by the different Values obtained for both the vacisek and the Cox, Ingersoll and Ross model. The results of the simulation showed that the cox, Ingersoll and Ross interest rate model provided a better fit of interest as compared to The Vasicek model.

Key words: Asset-liability management, C-3 Risk, Loss Reserves, Surplus, Stochastic models, Deterministic Models.

I. Introduction

The rate of interest rate plays a very crucial role in making investment decisions and management of risk in an insurance industry. In insurance, the underlying interest rate helps in determining the value of assets, liabilities and overall surplus. Kibanga (2019) highlighted several factors that affects the investment of insurance firms. Some of the factors he highlighted were; changes in interest rate/ interest rate fluctuations, rate of inflation and duration of the investment. Insurance companies needs to be aware of the future movement of interest rates in order to correctly value their insurance products. Under-valuation, brought about by over-approximation of future interest rate movement on assets than liabilities might expose the insurance company to the insolvency Risk. Over-valuation of insurance products is also a risk to the insurance company as it may ruin its reputation making potential customers to shy –off from its insurance products offered in the market.

Traditionally, valuing an Insurance policy was a very big challenge due to insufficient Information and knowledge on the movement of the interest rates. Generally, insurance surplus managers normally create a comparison on changes in surplus levels on every new strategy implemented to model the interest rate. Fluctuations in interest rates possess a very big challenge to insurance surplus managers. For this reason, they have to develop ways of studying the movements of interest rates in order to shield their surplus levels. The most commonly used models for predicting future interest rate movements for an insurance investment are the stochastic models.

Traditionally, Deterministic models were used but their usage was found unsatisfactory by most Actuaries in the insurance sector. Stochastic interest rate models proved to be superior in prediction of future interest rate movement. Stochastic models use a lot of past data on interest rate to predict The likelihood of future interest rate changes.

Deterministic models do not based on historical interest rate. Deterministic models assume the rate of interest rate follows a given specific pattern which repeats itself. In real situation, this pattern is hard to achieve due to the uncertainties that are involved in the predictions. Redington's approach of an insurance immunization strategy is based on deterministic approach. Most Insurance companies used his approach and others used an improved version of his approach to immunize their surplus against fluctuations of interest rates. The approach however had several limitations that made insurance companies to shift to stochastic approaches because of the random fluctuations of interest rates. This limitation was hard to realize because the insurance sector had not yet fully penetrated in most countries, especially Third world countries, where data about the historical interest rate is hard to get.

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In the insurance industry, an appropriate stochastic interest rate model needs to be chosen in order to immunize the insurance surplus against future losses. In insurance, the most popular stochastic interest rate models used are the vacicek and the Cox,ingersoll and Ross models.Other stochastic model that was commonly used in Insurance sector is the Hull and White model, which offers a good prediction of the movement of interest rate because the interest rate can easily be tracked. Its usage was found unsatisfactory because it lacks a definite formula for finding the value of insurance assets and liabilities.

This paper offers an extension of the work of Jennifer Wang et al (2002) on Model Risk and Surplus Management under A stochastic Interest Rate Process ,who applied simulation on the Vacicek (1977) model and Cox,Ingersoll and Ross interest rate model (1985),using historical calibration parameters of the models to determine the impact of not correctly valuing an interest rate on Insurance Assets, liabilities and Surplus.Here,we determine an appropriate stochastic Interest rate model for valuation of Insurance Assets ,Liabilities and Surplus by creating a comparison between the two short rate models, and using Geometric simulation on interest rate to recalibrate the parameters of the Vacisek and Cox ,Ingersoll and Ross Interest rate models.

II. Methodology

Desirable characteristics of a term structure model.

- The model should be free of arbitrage.
- Interest rates should not be negative.
- The risk free-rate and other interest rates should exhibit some form of mean-reverting behavior.
- The model should be easy to calculate the prices of bonds and derivative contracts.
- The model should produce realistic dynamics i.e it should reproduce features that are similar to the past features and with reasonable probability.
- The model should fit historical interest rate data adequately.
- The model should be easily calibrated to current market data.
- The model should be flexible enough to cope with a range of derivative contracts.

Standard Brownian motion. (Wiener process).

This is a stochastic process $\{B_{t;\, t} \geq 0\}$ with state space S=R (The set of all real numbers) and has the following defining properties.

- B_t has Gaussian increments, i.e. the distribution of $B_t - B_s$ is N (0, t-s).
- B_t has continuous sample paths, $t \rightarrow B_t$ i.e. the graph of B_t as a function of t does not have any breaks in it.
- B_t has stationary increments, i.e B_t - B_s is independent of $\{Br, r \leq s\}$ whenever s<t. Thus the changes in the value of the process over any two non-overlapping periods are statistically independent.
- B_o =0

2.2 The Vasicek (1977) interest rate model.

The Vacicek models the interest rate process, r (t), as;

$$dr_{(t)} = \alpha\ (\mu - r_{(t)})\ dt + \sigma dZ_t \qquad (2.1)$$

Where Z_t is a standard Brownian motion under Q which represents a random market risk. The α parameter takes a positive value and is the speed of the mean- reversion i.e. it is the momentum of the drift rate. α, μ and σ are constants.

$\alpha\ (\mu - r_{(t)})$ is the drift rate and it represents the expected change in interest rate at time t., $r_{(t)}$ is the interest rate at time t, μ is the speed-reversion level i.e. it is the mean of the long term interest rate, σ is the standard deviation of the interest rate process/volatility of interest rate.

Re-writing equation (2.1) in discrete form we get;

$$\Delta r_{(t)} = \alpha\ (\mu - r_{(t)})\ \Delta_t + \varepsilon_i \sigma_i \Delta_t. \qquad (2.2)$$

Where; $\Delta r_{(t)} = r_{t+1} - r_t$

ε_i is the standard normal variable.

According to Vasicek (1977), the current price, P(t), of one –dollar zero-coupon bond maturing in t periods, P(t), is;

$$P\ (t) = A(t) exp^{(-\beta(t) r)} \qquad (2.3)$$

Where r is the current level of interest rate.

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The parameters;

$\beta(t) = \frac{1-exp^{(-\alpha t)}}{\alpha}$, and (2.4)

$A(t) = \exp(\frac{(\beta(t)-t)(\alpha^2\beta-0.56^2)}{\alpha^2} - \frac{6^2\ \beta^2(t)}{4\alpha})$ (2.5)

Assumptions of the Vasicek (1977) model.

- The variation in the rate of interest rate for each period is constant.
- The interest rate process exhibits mean-reversion with a constant volatility

Limitations of the vacisek model.

- It allows for the possibility of negative interest rate which in real situation is hard to achieve.
- The model is a short-term single period model, which makes it difficult to apply the model for prediction of long-term interest rate movements.

Cox, Ingersoll, and Ross (1985) model.

The interest rate process r_t, is modeled as;

$dr_t = \alpha_0(\mu_0 - r_t)\,dt + \sigma_0\sqrt{r_t}\,dz$ (2.6)

Where α_0 ,μ_0 and σ_0 are constants and dz follows a standard Brownian motion.

The drift rate is:

$\alpha_0(\mu_0 - r_t)$

$\sigma_0\sqrt{r_t}$ is the standard deviation.

According to Cox, Ingersoll and Ross (1985), the current price of one-dollar zero coupon bond maturing in t periods, P (t) is;

$P(t) = P_0(t) = A_0(t)exp^{(-\beta_0(t)r)}$, (2.7)

Where r is the current level of interest rates.

Let

$\theta^2 = \alpha^2_0 + 2\sigma^2_0$ (2.8)

$A_0(t) = (\frac{2\theta o e^{t(\alpha 0+\theta 0)/2}}{(\theta o+\alpha o)(e^{t\theta o}-1)+2\theta o})^{2\alpha_o \mu_o / 6o^2}$ (2.9)

And

$\beta_o(t) = \frac{2(e^{t\theta o}-1)}{(\theta o+\alpha o)(e^{t\theta o}-1)+2\theta o}$ (2.9.1)

Assumptions.

- It assumes all interest rate remain positive.
- The volatility increases in line with the square root of r(t)
- The interest rate process exhibits mean reversion and with volatility that is inline to the level of interest rate.

Modelling the Cash flows of an insurance company.

Let M (t) denote the cash inflows of an insurance company and G(t) denote the cash outflows of the insurance company and t be future time period.

The Assets and Liabilities will satisfy the equations;

$A = \sum_{t=1}^{n} M(t)P^A(t)$, and (2.9.2)

$L = \sum_{t=1}^{n} G(t)P^L(t)$ (2.9.3)

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Where $P^A(t)$ and $P^L(t)$ are the current price of one-dollar zero-coupon bond maturing in t periods based on the interest rate process that is followed by the assets and liabilities respectively.

The surplus of an Insurance company is equal to;

$$S=A-L= \sum_{t=1}^{n} M(t)P^A(t) - \sum_{t=1}^{n} G(t)P^L(t) \quad (2.9.4)$$

If r_t^A and r_t^L are the rate of returns on assets and liabilities and also under the assumption that the insurance is interest rate sensitive and the company always maintains its interest rate for valuing liabilities as a fixed proportion for valuing its rate for valuing assets;

$r_t^L=k\, r_t^A$

Where k is a positive constant.

If the interest rate of assets follows the Vasicek's (1977) model, i.e $r_t^A = r_t$, then the interest rate for valuing liabilities would be;

$$dr_t^L = \alpha\,(k\mu - r^L_{(t)})\,dt + k\sigma dZ_t \quad (2.9.5)$$

This implies that the long run level and the volatility of the liability rate of return are proportional to those of the asset rate of return. The adjustment speed for the liability rate of return to its long-term level is the same as for the asset rate of return.

If the assets rate of return follows the Cox, Ingersoll and Ross's model, we have;

$$d\,r_t^L= \alpha_0\,(k\mu_0 - r_t^L)\,dt + \sqrt{k\sigma o}\sqrt{r_t L}\,dz \quad (2.96)$$

Just as in the Vasicek's model, the long term level of the liability interest rate is k times that of the asset return

III. Main Results.

Modelling the present values of future cash flows of an Insurance company.

We simulated the future cash flows of an Insurance Company over a ten-year period.

The surplus of the insurance company is given as;

Surplus=Total Assets-Total liabilities.

Assume that the insurance company models future cash flows using the Interest rate followed by the Vasicek model. The parameters of the vacisek model was estimated based on Geometric Brownian motion simulation on the price Index from 2000 to 2020.The results of the simulation gave us the following parameters of the Vacisek model:

Table 3. 1: Table showing Assumed Constants of the Vasicek model.

Assumed constants	Values
Current annual interest rate on assets, r_t	7.125%
Speed of the mean reversion, α	0.1559
Speed reversion level	0.0981
Volatility of the interest rate, σ	0.0250
Monthly interest rate	0.59375%
K	98.5%

In explaining our model, we also assumed that the Insurance Company knows the parameters of the Vasicek model.

The present Value of future cash flows is computed follows:

Present value of future Assets using the Vasicek model is given as;

$$A=\sum_{t=1}^{n} M(t)P^A(t) = \sum_{t=1}^{t=10} M(t)A(t)exp^{(-\beta_0(t)r)} = 6{,}133{,}640$$

The computed true value of future liabilities using the model is;

$$\sum_{t=1}^{n} M(t)P^L(t) = \sum_{t=1}^{t=10} M(t)L(t)exp^{(-\beta_0(t)r)} = 5{,}647{,}514$$

The present value of surplus is given by;

$$\text{Surplus=Assets-Liabilities}= \sum_{t=1}^{n} M(t)P^A(t) - \sum_{t=1}^{n} G(t)P^L(t) = 486{,}126$$

We also assumed that the Insurance company models future interest rates on its assets, liabilities and surplus with the assumptions given in the table below (according to the average price index from 2000 to 2020) parameters given below.

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Table 3. 2: Assumed constants of the Cox, Ingersoll and Ross interest rate model

Assumed constants	Values
Current annual interest rate on assets, r_t	7.125%
Speed of the mean reversion, α_0	0.3670
Speed reversion level	0.0908
Volatility of the interest rate, σ_0	0.07054
Monthly interest rate	0.59375%
k	98.5%

The present value of the future cash flows are re-calculated and the results obtained as shown below:

Present value of future Assets using the Cox Ingersoll and Ross Interest rate model is given as;

$A=\sum_{t=1}^{n} M(t)P^{A}(t) \quad =\sum_{t=1}^{t=10} M(t)A\,o(t)exp^{(-\beta_0(t)r)}$ =8,500,327

The computed true value of future liabilities using the model is;

$\sum_{t=1}^{n} M(t)P^{L}(t) \quad =\sum_{t=1}^{t=10} G(t)L\,(t)exp^{(-\beta_0(t)r)}$ = 5,655,200

The present value of surplus is given by;

Surplus=Assets-Liabilities= $\sum_{t=1}^{n} M(t)P0^{A}(t)$ -$\sum_{t=1}^{n} G(t)P^{L}(t)$ = 2,845,127

The present value of future cash flows when interest rate is modelled using Vasicek Model should equal the Present value of future cash flows when Cox,Ingersoll and Ross interest rate model is used

A Comparative study of the two models

The results of the simulation can be summarized on the table below.

Table 4. 3: Computed values of the model costs for our models.

Model	Assets	Liabilities	Surplus
Vasicek (1977) value	6133640	5647514	486126
Cox,Ingersoll and Ross (1985) interest rate value	8500327	5655200	2845127
Model cost (Differences)	2366687	7686	2359001
Percentage change (Vacisek)	38.585%	0.1361%	485.265%
Percentage change(Cox,Ingersoll and Ross model)	27.8423%	0.1359%	82.9137%

Model cost=Vasicek (1977) model value- Cox, Ingersoll and Ross (1985) Interest rate value.

Percentage change= (model cost/actual estimate value of the models)*100

IV. Conclusions and Recommendations.

Based on the above results and calculations, we can see that modelling the Cash flows of an Insurance Company using stochastic interest rate models is not the only solution to immunize an Insurance Company's Assets, liabilities and surplus against fluctuations in interest rates. We find that an Insurance Company needs to take so much care on the choice of the model for interest rate. We also see that the different assumptions and parameters of the two models yielded different values for the true value of Assets, liabilities and surplus. The Vasicek model yielded a higher Surplus value as compared to the Cox, Ingersoll and Ross interest rate model but it also yielded a higher percentage of model risk as compared to the Cox, Ingersoll and Ross Interest rate model.

A very significant part of the results depicts that this comparison of the two models has helped in quantifying the value of the model risk, which in most cases is negligible by most Insurance firms.

In practice, Insurance companies tries to minimize risks as much as possible. It would be prudent to use The cox, Ingersoll and Ross Interest rate as it resulted to a lower model risk as compared to the Vasicek model .The cox, Ingersoll and Ross Interest rate model does not also allow for the possibility of negative interest rate as compared to the Vasicek model which allows the possibility of having a negative interest rate.

Based on the results of this project and conclusions, we recommend the following;

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- That Insurance Asset liability managers using traditional approach in modeling future interest rates to adopt the use of stochastic interest rates to model their cashflows,as they are more superior than deterministic approaches.
- That Insurance asset liability managers using one stochastic interest rate model in modeling their cash flows should diversify and try other various stochastic interest rate models.
- Asset liability managers should try to explore other factors that makes the interest rate sensitive.
- Further research and Analysis should be done in order to provide a wider view of the accuracy of our models.

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