Predicting Mortality Rates and Longevity Using Cains -Blake-Dowd Model

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Abstract: Pension schemes and annuity providers frequently guarantee their retirement payouts until the retirees' deaths. As a result of longer life expectancies and declining rates of death in old age, trends in mortality and longevity have become evident. Academicians and actuaries have been forced to concentrate their research on mortality and longevity concerns in particular as a result of this. Instead of a provident fund, the new National Social Security Fund Act Number 45 of 2013 established a pension fund that is a requirement for every employee. Annuity service providers are exposed to the longevity risk when the scheme's participants retire. For pricing and reserving, appropriate modeling tools or projected life tables are required. In comparison to deterministic models, which were based on projected present values, stochastic models allow a variety of risk causes and components as well as pertinent effect on portfolio performance. The long term mean level of Longevity has become more uncertain exposing the annuity service providers such as assurance companies and states to the risk of uncertainty after retirement. Most industrialized countries' national security systems, pension plans, and annuity providers have revised their mortality tables to account for longevity risks due to decline mortality rates and rising life expectancy. Kenya is one of the developing nations that has seen a drop-in death rates and a rise in life expectancy recently. Since developing nations choose to take the longevity risk into account when pricing and reserving annuities because such long term mean level in mortality rates declines and increases life expectancy, particularly at retirement age, pose risks to annuity service providers and pension plans that has been pricing annuities based on mortality tables that do not take these trends into account. The stochastic aspect of mortality was ignored by earlier actuarial models used to estimate trends. The actuary will therefore likely be interested in knowing how the future mortality trend utilizing stochastic models affects annuity pricing and reserve. Demographers and actuaries have since employed a variety of stochastic methods to forecast mortality while examining a variety of stochastic model ranges. The CBD stochastic model, which was the first to take longer life expectancies into account, is now extensively used, and a number of expansions and adjustments have been suggested to stop the major characteristics of mortality intensity. The CBD model, developed by Andrew Cairn, David Blake, and Kevin Dowd, is being used in this study to fit mortality rates, forecast mortality trends, using least square method and then calculate projections for life expectancy. Regarding the longevity risk, we take into account the possibilities of computing annuity benefits by connecting the benefits to actual mortality and calculating the present value on annuities. The results of the study showed that, the CBD model can be used to forecast mortality rates where parameters estimating the CBD model are performed using the bivariate random walk (drift).

Keywords: Human Mortality database, defined Benefits, age-period-cohort, Defined contribution, Root Mean Square Error.

I. Introduction.

The long term mean level of Longevity has become more uncertain exposing the annuity service providers such as assurance companies and states to the risk of uncertainty after retirement.

Most industrialized countries' national security systems, pension plans, and annuity providers have revised their mortality tables to account for longevity risks due to decline mortality rates and rising life expectancy. Kenya is one of the developing nations that has seen a drop in death rates and a rise in life expectancy recently. Since developing nations choose to take the longevity risk into account when pricing and reserving annuities because such long term mean level in mortality rates declines and increases life expectancy, particularly at retirement age, pose risks to annuity service providers and pension plans that has been pricing annuities based on mortality tables that do not take these trends into account.

Longevity is a threat to pension funds and annuity service providers; it has been acknowledged. The mortality models were divided by Booth and Tickle (2008) into extrapolative models, explanatory models, and expectancies models. There have been more models proposed for explaining and estimating mortality as a result of recent improvements in actuarial methods, particularly in pensions and life mathematics. The models were conveniently surveyed and explained by (Pitacco, Denuit, Haberman, & Oliviera, 2009). It's still difficult to dynamically fit mortality rates and, thus, quantify longevity risk, especially in emerging nations. Prior work was based on the Lee and Carter in the year 1992 being one-factor model. However, this model is frequently used to give estimates and demographic projections that are quite accurate for academics and practitioners alike. This model was examined and a new model was developed by Halzoupoiz (1996) and Renshaw and Heberman (2003).

The cohort effect was recently taken into account in longevity modeling, which Lee and Carter's model lacked. For example, Currie (2006) offers an APC model after Renshaw and Haberman (2003) incorporated a cohort effect. The most recent proposals, made by CBD (2006b), find that all the issues with the Lee and Carter model can be resolved by including both a cohort impact

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and a quadratic age effect in their cairns-Blake-Dowd model. In this Research paper, we examine the stochastic aspect of mortality which was ignored by earlier actuarial models used to estimate trends. The actuary will therefore likely be interested in knowing how the future mortality trend utilizing stochastic models affects annuity pricing and reserve. The Cairns-Blake-Dowd (CBD) model take into account two factors to calculate the mortality rates. The two factors are employed in the CBD model to compute mortality rates. While the first component has an impact on mortality rate dynamics at older ages compared to younger ages, the second element has an equal impact on mortality rates at all ages. This model has been used in England and Wales, Spain, and Italy, to compute and forecast death rates.

II. Methodology

Mortality Assumption.

These mortality rate forecasts are used to calculate annuity prices and calculate pension liabilities. The mortality tables form the foundation for mortality assumptions. Since mortality rates and assumptions are essential to annuity pricing and reserving, the insurance and retirement benefits regulator in the majority of nations offers guidelines on their use.

The mortality assumption is one of the most important elements taken into account when estimating the life expectancy at birth. Estimating life expectancy will subsequently be used to compute the pension fund's and annuity providers' long-term liabilities. Long-term liabilities for both pension fund and insurance business are exaggerated if mortality assumptions are low. If the assumptions are too high, the pension plan's life expectancy will be overestimated, which will lead to an underestimation of the pension plans and annuity providers' responsibilities.

Inter-age dependency and heterogeneity

The degree of heterogeneity varies from population to population for a particular population. Heterogeneity results from a variety of observable elements, such as gender, age, occupation, and physiological parameters, as well as from aspects of the environment in which an individual lives, such as the climate, population, and dietary norms. greater socioeconomic status pensioners or insurance holders those with longer life expectancies or a propensity for experiencing lower mortality rates have greater life expectancies. Since females often have lower mortality rates than males, considerable differences also exist within the same socioeconomic status.

Closing the tables and blending

Empirical yearly death rates have erratic age distributions at advanced ages. Actuaries typically deduce the shape of the survival functions S(x) at higher ages from a few exogenous assumptions, which involves closing the mortality tables. In the past, death after 100 was not given much attention because it had a relatively minor effect on pensioners' residual life expectations (and thus, pensions). This is no longer the case due to recent advancements in longevity, and it is crucial to have better understanding of both mortality rate and risk of longevity for higher ages.

Initial Rate of Mortality

The mortality rate q_x which is the conditional probability of person aged x dying in the next year of age.

$q_x = \frac{d_x}{l_x}$ where;

d_x is the expected number of deaths at age x last b birthday among l_x lives aged x over the next year.

lx - represent the radix at start of each year (People alive).

Central death Rate

It is denoted by mx which represents the probability of dying between exact ages χ and χ +1 per person-year lived. Definition;

$T_x = \int_0^1 l_{x+t}\,dt = \int_x^{x+1} l_y\,dy = \int_x^{\infty} l_y\,dy - \int_{x+1}^{\infty} l_y\,dy = T_x - T_{x+1}$

Hence mx, can be written as;

$$m_x = \frac{d_x}{L_x} = \frac{\int_0^1 l_{x+t}\mu_{x+t}dt}{\int_0^1 l_{x+t}dt} = \mu_{x+1/2}$$

Force of Mortality

It is the death rate at exact time t for individuals aged x+t at time t. It is denoted by μx

$m_x = \lim_{h\to 0} 1/h * p[T \le x + h / T > x]$ $\mu x = \lim_{h\to 0} / h$

For the small h, $hq_x \simeq h.\mu_x$

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Expected Future Lifetime

This measures remaining time until death. It is given as $E\,[Tx]$;

$$\mathring{e}_x = \int_0^{w-x} t\,.\,p_x\mu_{x+t}dt = \int_0^{w-x} t\,.\left(\frac{\partial y}{\partial x} - tp_x dt\right) = -t * p_x|_0^{w-x} + \int_0^{w-x} p_x\,dt = \int_0^{w-x} p_x\,dt$$

The curtate future lifetime of a life aged x is, *Kx* = *Tx* represents integer part of future lifetime. Kx is given by e_x *E*[*Kx*]= $\sum_{k=1}^{w-x} kp_x$

Age Specific Death Rates

ASDR = $\frac{Totaldeaths \in aspecifiedagegroup}{Totaldeaths \in thesameagegroup}$ * 100,000

Cairns-Blake-Dowd Model

It is given as ;

$$\text{logit}(q_{x,t}) = K_t^{(1)} + K_t^{(2)}\,[x - \bar{x}] \quad (2.1)$$

where logit($q_{x,t}$)= $ln\left(\frac{q_{x,t}}{1-q_{x,t}}\right)$

where *x* is the age group (*x* = *x*1, *x*2, ..., *xp*), *t* is the period (*t* = *t*1, *t*2, ..., *tq*), $q_{x,t}$ is the mortality rate, which is the probability an individual at age group *x* in period *t* will die at intervals of time *t* and *t* + 1, *κt* (1) represents the intercept and *κt* (2) represents the slope, and $\bar{x}$ is the average of the age group. The parameters *κt* (1) and *κt* (2) can be estimated using the Least Square Method.

Estimation by Least Square Method

The parameters of CBD model can be estimated by minimizing the sum of squared residuals as follows:

$$S = \sum_{x=x_1}^{X_p} \sum_{t=t_1}^{t_p} x\left[ln\left(\frac{q_{x,t}}{1-q_{x,t}}\right) - k_t^{(1)} - (x - x)k_t^{(2)}\right]^2 \quad (2.2)$$

Derving Equation (2) against *κt* (1) and *κt* (2) gives;

$$\frac{\partial S}{\partial \kappa_t^{(1)}} = \sum_{x=x_1}^{X_p} 2\left[ln\left(\frac{q_{x,t}}{1-q_{x,t}}\right) - \kappa_t^{(1)} - (x - \bar{x})k_t^{(2)}\right] = 0 \quad (2.3)$$

$$\frac{\partial S}{\partial \kappa_t^{(1)}} = \sum_{x=x_1}^{X_p} 2\,(x - \bar{x})\left[ln\left(\frac{q_{x,t}}{1-q_{x,t}}\right) - k_t^{(1)} - (x - \bar{x})k_t^{(2)}\right] = 0 \quad (2.4)$$

By solving both equations, the least square estimates of *κt* (1) and *κt* (2) are obtained as follows:

$$\kappa_t\,(1) = \frac{\sum_{x=x_1}^{xp} ln\left(\frac{q_{x,t}}{1-q_{x,t}}\right) - k_t^{(2)} \sum_{x=x1}^{xp}(x-\bar{x})}{p} \quad (2.5)$$

and

$$\kappa_t\,(2) = \frac{p\sum_{x=x_1}^{xp} ln\left(\frac{q_{x,t}}{1-q_{x,t}}\right) - \sum_{x=x_1}^{xp} ln\left(\frac{qx}{1-q_{x,t}}\right) \sum_{x=x_1}^{xp}(x-\bar{x})}{p} \quad (2.6)$$

Substituting the results of equation (2.5) and (2.6) to equation (2.1) to compute the estimated mortality rate, will be compared with actual mortality rate to see the stability of the model.

Error of the estimation process can be calculated with the *Root Mean Square Error* as follows:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(yi - \hat{y}i)^2} \quad (2.7)$$

where *ŷi* is the estimated value, *yi* is the actual value, and *n* is sample size. If the error is small enough, then predicting the estimated value of parameters can be processed. We shall use the Bivariate random walk with drift to forecast the time series data.

Bivariate Random Walk (Drift)

Written as: *κt*(i) = *κt-1*(i) + μ(i) + ε(ι)t , t = t2, t3, …, tq (2.8)

Suppose *i* = 1, 2, then κ*t* = (*κt* (1), *κt* (2))′, μ = (*μ*(1), *μ* (2))′, and ε = (*εt* (1), *εt* (2))′, It can be given as follows;

kt – kt-1 = μ + εt t = t2, t3, …, tq

where μ is drift parameter and ε*t* ~ *N*(0, Σ). From Equation (2.8), it is obtained that

So that, in general, can be written into:

Y= M + ε

Y= M + ε

$$Y = \begin{bmatrix} \kappa_{t_2}^{(1)} - \kappa_{t_1}^{(1)} \kappa_{t_3}^{(1)} - \kappa_{t_2}^{(1)} \dots \kappa_{t_q}^{(1)} - \kappa_{t_{q-1}}^{(1)} \\ \kappa_{t_2}^{(2)} - \kappa_{t_1}^{(2)} \kappa_{t_3}^{(2)} - \kappa_{t_2}^{(2)} \dots \kappa_{t_q}^{(2)} - \kappa_{t_{q-1}}^{(2)} \end{bmatrix}$$

$$M = \begin{bmatrix} \mu_1 \mu_1 \mu_1 \\ \mu_2 \mu_2 \mu_2 \end{bmatrix}$$

Y, M, and ε is 2 × (q - 1) matrix. Equation (2.9) will be estimated with Ordinary Least Square (OLS),where GG = [1 1 ⋯ 1] is a constant $1 \times (q-1)$ vector, then the estimate u may be expressed as follow:: $\hat{u} = \frac{K_{t_q} - K_{t_1}}{q-1}$ $\hat{u}$ = (YG')(GG')$^{-1}$

Based on Equation 2. 8, it is obtained that:

$$K_{t_{q+m}} = K_{t_q} + m\mu + \sum_{j=1}^{m} \varepsilon_{t_{q+j}}$$

so the forecasting of the parameters $K_t^{(1)}$ and $K_t^{(2)}$ with t = tq (as in Equation (2.8)) for m periods ahead are:

Ktq+m = Ktq + mμ

Error checking is performed to evaluate the accuracy of forecasting results. This process can be done by using the RMSE in Equation (2.7).

Actuarial present Values

This $\ddot{a}_x^m$ the present value of an annuity of 1 p.a annum payable monthly in advance.

$$\ddot{a}_x^m = \sum_{t=0}^{\infty} \frac{1}{m} v^{\frac{t}{m}} \frac{t}{m} p_x \qquad (2.9.1)$$

This is expressed in terms of $\ddot{a}_x$ as follows;

Euler-McLaurin formula:

$$\int_0^{\infty} f(t)dt = \sum_{t=0}^{\infty} f(t) - \frac{1}{2} f(0) + \frac{1}{12} f'(0)$$

Woolhouse's formula: $\frac{1}{m} \sum_{t=0}^{\infty} f\left(\frac{t}{m}\right) = \sum_{t=0}^{\infty} f(t) - \left(\frac{m-1}{2m}\right) f(0) + \left(\frac{m^2-1}{12m^2}\right) f'(0)$

Assuming that ; $f(t) \rightarrow 0$ and $f'(t) \rightarrow 0$ as t tends to infinity

Using equation 3.5.1 to fit $f(t) = v^t \, {}_tP_x = exp\left(-\int_0^t (\delta + \mu_{x+r})\, dr\right)$ then

$f'(t) = -((\delta + \mu_{x+t})\ exp\left(-\int_0^t (\delta + \mu_{x+r})\, dr\right)$ therefore f(0) = 1 and f'(0) =- $(\delta + \mu_x)$ this gives $\ddot{a}_x^m = \ddot{a}_x - \left(\frac{m-1}{2m}\right) - \left(\frac{m^2-1}{12m^2}\right)(\mu_x + \delta)$

$$\ddot{a}_x^m = \ddot{a}_x - \left(\frac{m-1}{2m}\right)$$

III. Main Results

Source of data

Data sources utilized in modeling longevity risk are deaths of individuals in pension plans and annuity services providers. The UK's CMIB collects mortality data from those sources and information from pension schemes on insured lives. Although KRA collects this data, it is not widely accessible because it is never made public or published every year. Furthermore, usage of this data might lead to sampling issues because it is not the accurate representative of the whole population. HMD is where countries publish their mortality data. The most appropriate data is the entire population since it includes large number of individuals. Our analysis will be based on mortality data from United States obtained from the HMD via the demography package's dedicated function. The data in HMD consist of sex , age and year.

Fitting the Model

U.S.A mortality rates data ranges from 1950-1955 to 2015-2020 with age groups 0, 1–4, …, 80–84 years old. Let age group (0) years old be x1, (1–4) years old with x2, (5–9) years old with x3, ..., 80–84 years old with x18. Let again x be the midpoint value of each age group that is 0, 2.5, 7, …, 82 years old.

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The value of (x - $\bar{x}$) has a considerable impact on logit ($q_{x,t}$) in the CBD model, hence the age group utilized in the parameter estimate method influences the mortality rates. As a result, simulations are run during the parameter estimation process to obtain the best estimated results. To obtain best estimated results, simulation in the process parameter estimation divides 18 different age groups into multiple groups. Parameter estimate simulations were performed using the age group x1-x18 and by dividing the 18 age groups into three different groups, that is x1-x6 (children and adolescents), x7-x13 (adults), and x14-x18 (elderly). Equations 2. 5 and 2.6 are used to calculate parameter estimations.

Table 3.0.The estimated values of $\hat{K}t(1)$ and $\hat{K}t(2)$

T	Periods	$\hat{K}t(1)$	$\hat{K}t(2)$
t1	1950-1955	-2.415646996	0.037325948
t2	1955-1960	-2.507819312	0.039018362
t3	1960-1965	-2.599202081	0.040700171
t4	1965-1970	-2.689945927	0.042376603
t5	1970-1975	-2.789380200	0.044222561
t6	1975-1980	-2.913925914	0.046647877
t7	1980-1985	-3.034071316	0.048944461
t8	1985-1990	-3.114601821	0.051105996
t9	1990-1995	-3.205308913	0.053500361
t10	1995-2000	-3.297892093	0.05770258
t11	2000-2005	-3.337483154	0.05770258
t12	2005-2010	-3.406147813	0.059173814
t13	2010-2015	-3.478742245	0.06175235
t14	2015-2020	-4.056432455	0.06246754

To generate an approximated value of $q_{x,t}$ Table 2 will be replaced into Equation 2. 1. The anticipated value of $q_{x,t}$ will be compared against the actual value of $q_{x,t}$ to determine the model's appropriateness.

The RMSE computation is performed using Equation (2.7) to determine the error from the estimated $q_{x,t}$ findings.

Table3.1. The estimated values of $\hat{K}t(1)$ and $\hat{K}t(2)$ with three partition groups

		x1–x6		x7–x13		x14–x18	
t	period	$\hat{K}t(1)$	$\hat{K}t(2)$	$\hat{K}t(1)$	$\hat{K}t(2)$	$\hat{K}t(1)$	$\hat{K}t(2)$
t1	1950-1955	-3.143545145	-0.11302412	-3.128116411	0.048260644	-0.544712036	0.114542622
t2	1955-1960	-3.293842153	-0.10660832	-3.196490027	0.049781847	-0.600452903	0.114680792
t3	1960-1965	-3.442797063	-0.10024799	-3.264521541	0.051301960	-0.655440860	0.114845829
t4	1965-1970	-3.590694945	-0.09388190	-3.332311770	0.052827236	-0.709734924	0.115020959
t5	1970-1975	-3.752810960	-0.08685924	-3.406916885	0.054498474	-0.768711931	0.115218461
t6	1975-1980	-3.945040132	-0.08564213	-3.531508946	0.058461945	-0.811972607	0.116492851
t7	1980-1985	-4.129440187	-0.08439366	-3.651459539	0.062164793	-0.855285159	0.117658517
t8	1985-1990	-4.284653652	-0.07803920	-3.704331129	0.064204521	-0.884918592	0.117266797
t9	1990-1995	-4.460861394	-0.06942212	-3.758352374	0.066049331	-0.924385090	0.116666379
t10	1995-2000	-4.640922339	-0.05945270	-3.808333383	0.067620892	-0.971637992	0.115677701
t11	2000-2005	-4.764786467	-0.04540144	-3.782162694	0.067286918	-1.002167823	0.113028112
t12	2005-2010	-4.912600833	-0.03526591	-3.803541879	0.068413398	-1.042052494	0.111368643

t13	2010-2015	-5.065014774	-0.02820669	-3.856003369	0.070077092	-1.077402119	0.110681393
t14	2015-2020	-5.165225531	-0.02920669	-3.896003369	0.072077092	-1.97402119	0.100681393

Table 3.2 Shows RMSE values of estimated $q_{x,t}$ for entire age groups.

Age Group	RMSE	Age Group	RMSE	Age Group	RMSE	Age Group	RMSE
0	0.093301	20–24	0.011492	45–49	0.036859	70–74	0.099627
1–4	0.047992	25–29	0.01662	50–54	0.038031	75–79	0.193946
5–9	0.000729	30–34	0.02180	55–59	0.035057	80–84	0.300001
10–14	0.008453	35–39	0.02687	60–64	0.006523		
15–19	0.009628	40–44	0.03201	65–69	0.030089		

Table 3.3. RMSE value of estimated $q_{x,t}$with partition into three groups.

Age Group	RMSE	Age Group	RMSE	Age Group	RMSE	Age Group	RMSE
0	0.047181	20–24	0.00686	45–49	0.003039	70–74	0.007628
1–4	0.014864	25–29	0.00182	50–54	0.000558	75–79	0.003775
5–9	0.015614	30–34	0.00021	55–59	0.008138	80–84	0.009236
10–14	0.01004	35–39	0.00160	60–64	0.00493		
15–19	0.000655	40–44	0.00267	65–69	0.00523		

Forecasting Parameters and Mortality Rates

Parameters $K_t^{(1)}$ and $K_t^{(2)}$ are forecasted using the Bivariate Random Walk (Drift). From the results in Table 2, the estimated values of $K_t^{(1)}$ and $K_t^{(2)}$ by using Equation (2.9.1) we obtain μ for each group *x*1–*x*6, *x*7–*x*13 and *x*14–*x*18 as follows:

μ_a = $(-0.1601225, 0.00706812)$, μ_b = (-0.06065725, 0.001818),

μ_c = (-0.04439084, -0.00032)

Then, the forecasting parameters $K_t^{(1)}$ and $K_t^{(2)}$ will be calculated for the period 1955–1960, …, 2015–2020 using Equation (2.9.1). As can be shown in Table 3.2.

Next, based on the forecasted value of parameters in Table 3.4 and estimated parameters in Table 3.2 for the period 1955–1960 …, 2015–2020, the RMSE computation will be done using Equation (2.7) in order to find out the error from predicted results of $K_t^{(1)}$ and $K_t^{(2)}$.As shown in table (3.3)

Clearly the it can be seen that values of $K_t^{(1)}$ and $K_t^{(2)}$are quite small for each group which means that predicting method with bivariate random walk (Drift) is better enough to be use to predict $K_t^{(1)}$ and $K_t^{(2)}$. The predicting results of this parameter will be then replaced in Equation (2.1) to get the predicted value of $q_{x,t}$ and then it will be further examined with the actual value of $q_{x,t}$ to determine the suitability of forecasting method performed

Table 3.4. Predicted values of parameters κt(1) and κt(2) for the period 1955-1960 …, 2015-2020.

		x1–x6		x7–x13		x14–x18	
t	period	$\hat{K}$t(1)	$\hat{K}$t(2)	$\hat{K}$t(1)	$\hat{K}$t(2)	$\hat{K}$t(1)	$\hat{K}$t(2)
t2	1955-1960	-3.30366761	-0.10595600	-3.188773658	0.0500787	-0.58910288	0.11422085
t3	1960-1965	-3.45396462	-0.09317987	-3.325178787	0.0531200	-0.69983170	0.11452406
t4	1965-1970	-3.60291953	-0.09317987	-3.325178787	0.0531200	-0.69983170	0.11452406
t5	1970-1975	-3.75081741	-0.08681378	-3.392969017	0.0546453	-0.75412576	0.11469919
t6	1975-1980	-3.91293343	-0.07979112	-3.467574131	0.0563165	-0.81310277	0.11489669
t7	1980-1985	-4.10516260	-0.07857401	-3.592166193	0.0602800	-0.85636345	0.11617108

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t8	1985-1990	-4.28956266	-0.07732554	-3.712116786	0.0639828	-0.89967600	0.11733675
t9	1990-1995	-4.44477612	-0.07097108	-3.764988375	0.0660226	-0.92930943	0.11694503
t10	1995-2000	-4.62098386	-0.06235400	-3.819009621	0.0678674	-0.96877593	0.11634461
t11	2000-2005	-4.80104481	-0.05238458	-3.868990630	0.0694389	-1.01602883	0.11535593
t12	2005-2010	-4.92490894	-0.03833332	-3.842819941	0.0691050	-1.04655866	0.11270634
t13	2010-2015	-5.07272330	-0.02819779	-3.864199126	0.0702314	-1.08644333	0.11104687
t14	2015-2020	-5.17271380	-0.02830780	-3.87419915	0.0716319	-1.1860435	0.11004713

Table 3.5. RMSE values of forecasted κt(1) and κt(2).

	$x1$–$x6$		$x7$–$x13$		$x14$–$x1$	
period	$\hat{K}t(1)$	$\hat{K}t(2)$	$\hat{K}t(1)$	$\hat{K}t(2)$	$\hat{K}t(1)$	$\hat{K}t(2)$
RMSE	0.01871021	0.003403717	0.03800260	0.00106784	0.00956901	0.001064359

Table 3.7. Predicted values of κt (1) and κt (2) for the period 2020-2025, and 2025-2030.

Age Group	RMSE	Age Group	RMSE	Age Group	RMSE	Age Group	RMSE
0	0.04483099	20-24	0.00631994	45-49	0.00325275	70-74	0.007919307
1-4	0.01098748	25-29	0.00172644	50-54	0.00139707	75-79	0.004234615
5-9	0.01353896	30-34	0.00069549	55-59	0.00779937	80-84	0.009890479
10-14	0.00895576	35-39	0.00185672	60-64	0.00490045		
15-19	0.00065824	40-44	0.00289712	65-69	0.00595395		

Table 3.9. The outcomes of forecasted mortality rates

Age		Actual Value			Forecasted Value	
Group	1950-1955	…2005-2010	2010-2015	2015-2020	2020-2025	2025-2030
15-19	0.01795	… 0.00641	0.00588	0.004626	0.004141	0.003707
20-24	0.02238	… 0.00854	0.00790	0.004164	0.003861	0.00358
25-29	0.02338	… 0.00924	0.00857	0.006726	0.006163	0.005647
30-34	0.02631	… 0.01087	0.01013	0.009607	0.008885	0.008217
35-39	0.03133	… 0.01421	0.01335	0.013706	0.012794	0.011943
40-44	0.03858	… 0.01957	0.01857	0.019519	0.018391	0.017328
45-49	0.04782	… 0.02872	0.02762	0.027728	0.026371	0.025079
50-54	0.06641	… 0.04304	0.04163	0.039252	0.037681	0.036171
55-59	0.09348	… 0.06526	0.06347	0.055293	0.053574	0.051907
60-64	0.16136	… 0.10828	0.10584	0.097494	0.093937	0.090484
65-69	0.23886	… 0.16533	0.16069	0.157946	0.152339	0.146896
70-74	0.36295	… 0.25078	0.24386	0.245679	0.237545	0.229599
75-79	0.50507	… 0.37540	0.36593	0.361238	0.350693	0.340292
80-84	0.65154	… 0.53017	0.52033	0.495451	0.483555	0.471677

Based on Table 3.4 and Table 3.7, all groups have the best estimation and forecasting results of $q_{x,t}$. So, predicting mortality rates will be carried for all ages by replacing the predicted values of κt (1) and κt (2) from Table 3.8 to Equation (2.1).

Executing Actuarial Projection

I used the following formula calculate life expectancy and the actuarial present values;

$ln(\hat{\mu}_{x,t}) = \hat{a}_x + \hat{b}_x\hat{k}_t$ and $\hat{p}_{x,t} = exp(-\hat{\mu}_{x,t})$

We computed the actuarial present value of a_{65}^{12} for the total population as can be shown in the table below.

Table 3.9.1: the actuarial present value of a_{65}^{12} for the total population

Cohort for Year	Life expectancy	Present values
1950	55.62	6.10
1960	65.03	6.53
1970	72.49	6.98
1980	77.68	7.48
1990	80.01	7.68
2000	82.88	8.20
2010	84.39	8.58
2020	86.02	8.78
2030	89.99	9.20

There is an increase of annuity due to lower mortality rates as well as increase of life expectancy actuarial present value results show that annuities have increased through time as a result of lower mortality rates and increased life expectancy. As a result, the amount that annuity providers should pay to individuals must be reduced in order to avoid overpaying annuitants. Longevity risk arises because general degree of mortality change is uncertain at the time annuities are purchased.

IV. Conclusions and recommendations

Conclusions

It is evident that the best estimate of mortality rate is by using the CBD model, which is obtained by breaking down eighteen age-groups(18) into three different groups. Parameter predicting results using the bivariate random walk(drift) shows the long term mean level as the actual parameter value. Forecasting qx,t is carried out for ages 15–19, 20–24, 25–29, ..., 80–84 years old because it has the least Root Mean Square Error(RMSE) value as shown in (Table 3.4 and Table 3.7). The mortality rates for the next two periods have a downward long term mean level as shown in table 9 which is consistent from the period from the 1950–1955 …, 2010–2015. Thus, the CBD model can be used to forecast mortality rates where parameters estimating the CBD model are performed using the bivariate random walk(drift).

Recommendations

Since the risk of longevity exists in both pension schemes and annuity service providers that is assurance companies, therefore, I urge the management of longevity risk to implement these opinions. Therefore, individuals are encouraged to work for longer periods in order to save more.

Longevity risk being a major concern for both annuity service providers and pension schemes, we advise future academicians to use other models in place of the CBD to model and predict mortality rates, subsequently they measure longevity risks. This will address CBD's limitations.

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